Chapter 4 Section 2

MA1020 Quantitative Literacy

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The Method of Sealed Bids

Circumstance: Any number of players, n, are to share any number of items, with the possibility of monetary compensation to ensure fairness.

1. All players independently submit bids, each stating a monetary value for each item–that is, they make sealed bids.
2. Each item goes to the high bidder, and that player contributes the dollar amount of his or her bid to a compensation fund.
3. From the compensation fund, each player receives $\frac{1}{n}$ of his or her bid on each item.
4. Any money left in the compensation fund is distributed equally to all players.

The Method of Points for Three Players and Three Items

Circumstance: Three players are to decide who gets which of three items.

1. Each player assigns pts to each item so that the pts total 100 for that player.
2. List all six possible arrangements of players and items, along with the point assignments made by each player for that item.
3. For each arrangement, note the smallest number of points assigned to an item by a player. If there is exactly one arrangement for which the smallest number is as large as possible, that is the arrangement to use. If more than one arrangement has the maximum smallest number, in other words, if there is a tie, keep only those arrangements with the same maximum smallest number and move on to the next step.
4. For each arrangement kept in the prior step, note the middle (second-to-largest) number among the three assigned by the players. If exactly one arrangement has a maximum middle number, that is the arrangement to use. If there is more than one such arrangement (another tie), keep all those arrangements with the same maximum middle number and move on to the next step.
5. For each arrangement kept in the prior step, not the largest number of the three assigned by the players. Select any arrangement for which that largest

Adjusted-Winner Procedure for Two Players

Circumstance: Two players are to divide any number of items fairly. Items may be discrete or continuous, and ownership of some items may be shared.

1. Each player assigns points to each item so that the points total 100 for that player.
2. Each player tentatively receives those items to which he or she assigned more points than the other player, and we add the points corresponding to those items to the player's total. If the two players have assigned the same number of points to an item, the item goes to the player who has the smallest point total based on the other items that have already been tentatively distributed.
3. If the players' point totals are not equal and, say, player X has more points than player Y, select the item currently assigned to player X for which the ratio

$$\frac{\text{number of points assigned by player } X}{\text{number of points assigned by player } Y}$$

Adjusted-Winner Procedure for Two Players

4 Reexamine the players' point totals and decide what to do next based on whether or not their totals are equal. One of the following three cases will fit the situation.

   1. If the players now have exactly the same point total, then we are done.
   2. If player X still has more points than player Y, repeat the prior step.
   3. If player Y now has more points than player X, move a fraction of the item last moved from X to Y *back* to X to achieve equality. Use the formula on the next slide to calculate what fraction of the item should be returned to player X:

Formula

q = fraction of the item in question to be moved from player Y back to player X

T_X = player X's point total (not including the item in question)

T_Y = player Y's point total (not including the item in question)

P_X = number of points player X assigned to the item in question

P_Y = number of points player Y assigned to the item in question

$$q = \frac{T_Y - T_X + P_Y}{P_X + P_Y}$$