Chapter 6 Section 1 MA1020 Quantitative Literacy

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Königsberg Bridge Problem



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- Euler Circuit Problems
- Graph Theory
- Graph
- Vertices
- Edges
- Loop
- Adjacent Vertices

Definition

The degree of a vertex in a graph is the total number of edges at that vertex. If a loop connects a vertex to itself, the degree of that vertex is 2.

Theorem (Relationship Between Edges and Sum of Degrees in a Graph.) If d is the sum of the degrees of all the vertices in a graph and e is the number of edges in a graph, then d = 2e.

Paths & Circuits

- Path
- Circuit
- Euler Path
- Euler Circuit

Connected Graphs

- Connected
- Disconnected
- Components



Theorem

Any graph must have an even number of vertices of odd degree.

Theorem (Euler's Theorem.)

For a connected graph:

- **1** If the graph has no vertices of odd degree, then it has at least one Euler circuit (which is also an Euler path), and if a graph has an Euler circuit, then it has no vertices of odd degree.
- 2 If the graph has exactly two vertices of odd degree, then it has at least one Euler path but does not have an Euler circuit. An Euler path in the graph must start at one of the two vertices of odd degree and end at the other.
- 3 If the graph has four, six, eight, or larger even number of vertices with an odd degree, then it does not have an Euler path.

Fleur's Algorithm for Finding An Euler Circuit or Euler Path

- Make a copy of the original graph and label it "unnumbered edges." Make a second copy of the vertices without the edges of the original graph and label it "numbered edges."
- 2 Choose any vertex of the original graph with unnumbered edges and highlight it as a selected vertex.
- 3 Consider all edges connected to the selected vertex. Remove one edge. Give it the next number (starting with 1) and shift it to the graph with numbered edges. Do not choose an edge that leaves behind a disconnected graph (that is, do not remove a bridge), unless the only edge attached to the selected vertex is a bridge. Give the shifted edge a number to keep track of the order in which the path is being constructed.
- 4 If the edge removed was the last remaining edge in the whole graph with unnumbered edges go to step six.
- 5 If the edge you removed was not the last remaining edge, highlight the vertex on the other end of the removed edge as the new selected vertex.

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