MA1020 Final Exam Fall 2006

- (b) Explain mathematically, how you know for certain your answer to part (a) does indeed tile the plane. [2] Solution. A square has interior angle measure of 90°. Therefore, exactly four squares will fit around a vertex to form an edge-to-edge regular tiling of the plane.
 2. List the first 8 terms of the Fibonacci Sequence. [2] Solution. 1, 1, 2, 3, 5, 8, 13, 21 □
 3. (a) Name a real world example of the Golden Ratio.
 Parthenon [2]
 (b) Describe *in detail* how the Golden Ratio is applied in your answer to part (a). [2] Solution. The length divided by the width of the Parthenon is approximately the Golden Ratio. □
- 4. In a poll of 3000 residents of a state it is found that 600 are in favor of naming the cow the state animal.
 - (a) What percent of the sample is in favor? 20% [2]
 - (b) How many resident votes are needed for a majority. 1501 [2]
- 5. There are three candidates running for president of the senior class: Peter, Carmen, and Shawna. Voters mark their ballots to indicate their first, second, and third choices (only ballots with all three choices marked are valid). The results are summarized as follows:

| Candidate | 1st-Place Votes | 2nd-Place Votes | 3rd-Place Votes |
|-----------|-----------------|-----------------|-----------------|
| Peter | 33 | 68 | 34 |
| Carmen | 53 | 28 | 54 |
| Shawna | 49 | 39 | 47 |

Who is elected president using the plurality method?

1. (a) Name a regular polygon which will tile the plane.

Carmen [2]

6. True or False? There exists a voting system which satisfies all fairness criteria. False [1]

Square [2]

- 7. Consider the weighted voting system [7|5,3,2] with voters P_1 , P_2 , and P_3 .
 - (a) List all winning coalitions. [4] Solution. $\{P_1, P_2\}$; $\{P_1, P_3\}$; and $\{P_1, P_2, P_3\}$.
 - (b) How many votes are needed to pass a motion. 7 [2]
 - (c) Does anyone have veto power? If so, who? Yes, P_1 [2]
- 8. Complete the sentence. "A division is considered **envy-free** if each of the n players feels that he or she ..." [2]

Solution. ... has received at least $\frac{1}{n}$ of the total value and that no other player has a share more valuable than his or her own.

9. Fred, Greg, Hilde, and Irma inherited some lake property, and decided to use the "method of sealed bids" to divide it. Their bids on the property are given below:

| Fred | Greg | Hilde | Irma |
|-----------|-----------------------|-----------|-----------------------|
| \$500,000 | \$720,000 | \$680,000 | \$700,000 |

(a) Who gets the property?

Greg [2]

(b) After each person receives $\frac{1}{4}^{\text{th}}$ of his or her bid from the compensation fund, how much money is leftover and needs to be distributed?

\$70,000 [2]

- 10. In the last-diminisher method of fair division, what action should a chooser perform if he or she finds the piece of "cake" being examined to be less than a fair share?
 - (a) Approve the piece and move on to the next chooser.
 - (b) Trim the piece to a smaller size that is still a fair share and return the excess to the main body of the "cake."
 - (c) Pass the "cake" back to the player who last approved it.
 - (d) Restart the entire division process.
 - (e) None of the above.

(c) [2]

11. A western equipment company has facilities in San Francisco (SF), Butte (B), Denver (D), Salt Lake City (SLC), and Los Angeles (LA). The railroad distances between these cities are listed in the following table.

| Cities | SF | В | D | SLC | LA |
|--------|------|------|------|-----|------|
| SF | - | 1180 | 1370 | 820 | 470 |
| В | 1180 | — | 890 | 430 | 1220 |
| D | 1370 | 890 | — | 570 | 1350 |
| SLC | 820 | 430 | 570 | — | 780 |
| LA | 470 | 1220 | 1350 | 780 | _ |

(a) Draw a weighted graph representing the railroad connections between the company's facilities.
[3]

Solution.



(b) Remove redundant edges until the graph no longer contains a circuit. Draw the resulting subgraph. (Many answers are possible.) [3] Solution.



12. Consider the path represented by the sequence ABDAEDCBE of edges on the graph below.



Explain why the path is not an Euler circuit. [4]Solution. The path is not a circuit; it does not return to the starting point.

13. Consider the graph below.



Explain why the dashed edges are not

(a) a spanning tree. [2]

Solution. The dashed edges do not span the vertices; they do not hit pass through all the vertices. $\hfill\square$

- (b) a Hamiltonian circuit. [2]Solution. The dashed edges do not pass through all the vertices.
- 14. Consider the order-requirement digraph below.



(a) At time 0 which tasks are ready?

 T_1 and T_4 [1]

(b) Use the digraph to schedule the 6 tasks on two processors using the priority list T₁, T₆, T₃, T₅, T₄, T₂. (Display the schedule with a Gantt chart.) [5] Solution.



(c) Is this schedule optimal? If so, how do you know? [2]
 Solution. Yes, the schedule is optimal. The finishing time for the project is equal the the critical time.

- (d) Find the critical path and its length. [2] Solution. The critical path is $T_4 \to T_5 \to T_3$ and has length 18.
- 15. Consider the following table with tasks and completion times for cooking lasagna.

| Task Number | Task | Completion Time |
|-------------|---|-----------------|
| T_1 | Brown the meat | 10 min |
| T_2 | Add the tomato ingredients, garlic and basil to meat | $5 \min$ |
| T_3 | Simmer the meat sauce | $30 \min$ |
| T_4 | Boil the water | $10 \min$ |
| T_5 | Cook and rinse noodles | $10 \min$ |
| T_6 | Beat the eggs | $2 \min$ |
| T_7 | Mix beaten eggs with ricotta and parmesan cheese and seasonings | $6 \min$ |
| T_8 | Slice the mozzarella cheese | $7 \min$ |
| T_9 | Assemble the layers | 8 min |
| T_{10} | Bake | $30 \min$ |

Create an order-requirement digraph for the lasagna project. [4] Solution.



16. Consider the following histogram.



Describe the distribution using terms from chapter 8. [3] Solution. The graph is skewed left with a gap and an outlier.

17. The following bar graph displays the total number of turkeys produced in the US from 1975 through 2000.



(a) Which 5-year period showed the greatest change in turkey production?

1985 to 1990 [2]

(b) Create a line graph of the turkey-production data, using a vertical scale that emphasizes the changes in turkey production over time. [3] Solution.



- 18. The city council would like to determine how local voters feel about eliminating metered parking downtown. A survey is taken of adults who are shopping downtown on one afternoon.
 - (a) What is the population in this case? [2] Solution. Voters in a certain city.

| (b) | What is the sample? [2] | |
|-----|---|--|
| | Solution. Adults shopping downtown on a particular afternoon. | |
| (c) | What is the variable of interest? $[2]$ | |
| | Solution. The opinion local voters about eliminating metered parking. | |
| 1 | | |

- (d) Describe any bias in the survey. [2]
 Solution. The shoppers are not the only ones who may use the meters. For example, doing the survey this way would not take into account the opinions of those that live nearby and have to park on the streets.
- 19. Create a box-and-whisker plot for the following data which gives the monthly rainfall in inches for Portland, OR. [4]

Solution. The five number summary is $\{0.46, 1.54, 2.68, 4.55, 6.16\}$. Therefore the box-and-whisker plot is as follows.



20. A company would like to find out how the number of defective items produced varies between the day, evening, and night shifts. The following table shows the results of a sample of items taken from each shift.

| | Day | Evening | Night |
|--------------|-----|---------|-------|
| Defective | 24 | 28 | 47 |
| Nondefective | 279 | 224 | 165 |

If an item is picked at random, find the **probability** that

(a) the item is not defective, given that it came from the day shift.

0.921 [3]

(b) the item was produced by the night shift, given that it was not defective.

0.247 [3]

- 21. A penny, a nickel, and a dime (all fair) are flipped at the same time and it is recorded whether each coin lands head or tails.
 - (a) Find the sample space. [2] Solution. $\{HHH\}$, $\{HHT\}$, $\{HTH\}$, $\{HTT\}$, $\{THH\}$, $\{THT\}$, $\{TTH\}$, and $\{TTT\}$. \Box

- (b) What is the probability that all three coins land heads?
 (c) What is the probability that part (b) does not occur?
 ⁷/₈ [2]
- 22. A data set is represented by a normal distribution with a mean of 206 and a standard deviation of 22. For a data value of 218, how many standard deviations above or below the mean is it?

0.545 standard deviations above the mean [3]

23. The average length of time, per week that students at a certain university spend on homework is normally distributed with a mean of 23.7 hours and a standard deviation of 2 hours. Mike tells his parents that he spends more time studying than 97.5% of the students on campus. At least how many hours per week must Mike spend on homework for this to be true?

27.7 hours [3]