MA1020 Quantitative Literacy – Test 1

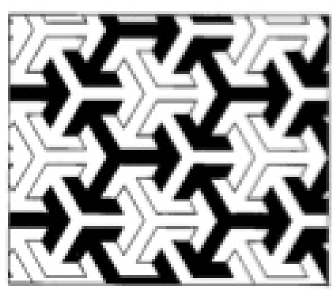
Solutions

October 6, 2006

1. Can a regular pentagon tile the plane? Explain your answer by using angle measures.
   Solution. No. The vertex angle measure of a regular pentagon is $\frac{(5-2)180}{5} = 108°$. If three pentagons surrounded a vertex, there would be $360° - 324° = 36°$ remaining, which is not enough space to squeeze a fourth pentagon. □

2. Describe in as much detail as you can a real world application of the golden ratio.
   Solution. you needed to provide an accurate description which included how the golden ratio is applied. Examples: Architecture–Parthenon, UN Headquarters; Art–da Vinci, Botticelli, Vermeer; Everyday objects–TVs, cereal boxes, credit cards. □

3. Consider the following two-dimensional pattern, and assume it continues indefinitely in all directions.



   What kinds of symmetries does it have?
   Solution. The pattern includes rotation, reflection and translation symmetries. □

4. An election features three candidates (A, B, and C). Each voter ranks the candidates in order of preference. The results are below:

	Number of votes		
Ranking	18	12	11
1^{st}	A	B	C
2^{nd}	B	C	B
3^{rd}	C	A	A

Who wins the election if the plurality with elimination method is used?
Solution. In the first round, C is eliminated. Then B is the winner in the second round. □

5. Which voting method that we studied *must always* satisfy the head-to-head criterion?
Solution. The pairwise comparison voting method always satisfies the head-to-head criterion. □

6. Consider the weighted voting system $[7|1,5,6]$. Does P_3 have veto power? Support your answer.
Solution. The winning coalitions are $\{P_1, P_3\}$, $\{P_2, P_3\}$ and $\{P_1, P_2, P_3\}$. P_3 is a critical voter in all of these coalitions, so P_3 does have veto power. □

7. Alice and Bob are dividing up a box of ten chocolates valued at \$10. Half the box consists of chocolate toffee and the other half is cream-filled chocolate. Bob likes toffee three times as much as he likes cream-filled. What monetary value would Bob place on each piece of toffee?
Solution. Bob would place a value of \$7.50 on the half that is toffee. Thus, he values each piece of toffee at \$1.50. □

8. In the last-diminisher method of fair division, what action should a chooser perform if he or she finds the piece of the "cake" being examined to be worth more than a fair share?

   (a) Approve the piece and move on to the next chooser.

   (b) Trim the piece to a smaller size that is still a fair and return the excess to the main body of the "cake."

   (c) Divide the piece into smaller pieces and let the other players decide which of the smaller pieces is a fair share.

   (d) Restart the entire division process.

   (e) None of the above.

Solution. (b), trim the piece to a smaller size that is still a fair share and return the excess to the main body of the "cake." □

9. For each of the following, explain which fair-division problem applies: continuous, discrete, or mixed fair-division problem. Explain your reasoning.

   (a) The money in a large savings account
   Solution. Continuous. Money can be divided in any way. □

   (b) The instruments and equipment used by a rock band
   Solution. Discrete. Instruments and equipment may not be divided individually. □

   (c) An inheritance consisting of a trust fund, a house, and several cars
   Solution. Mixed. A house and cars are discrete but a trust fund is continuous. □

10. Chris and Daniel are splitting up a pizza. The pizza is half cheese and half sausage. Chris likes sausage three times as much as cheese. Daniel likes cheese pizza twice as much as sausage. They agree to use the divide-and-choose method for two players, with Chris as the divider. Give an example of a fair division that Chris might make, and then indicate which option Daniel would choose and explain why.
Solution. First Chris will cut the pizza into 6 evenly sized pieces, where three are all cheese and three are all sausage. Next he will place two sausage on plate one and one sausage and three cheese on plate two. Daniel will choose plate two because he likes cheese more than sausage. □

11. Three children are arguing over which seat in the minivan they get to have – the front passenger seat, the rear driver seat, and the rear passenger seat. The mom has them assign 100 points to each of the three seats, and the results are as follows:

	Child		
Seat	A	B	C
Front passenger	36	50	30
Rear driver	40	40	50
Rear passenger	24	10	20

Using the method of points, determine which child will sit in which seat.
Solution. First, list all the six possible seating arrangements.

Front Passenger	Rear Driver	Rear Passenger
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Then consider the smallest number in each arrangement. They are 20, 10, 20, 24, 10, and 24, respectively. The two arrangements with the smallest number of 24 would maximize the satisfaction. Since there is a tie, consider the next smallest numbers. These are 30 and 50. 50 is much more desirable for maximizing the second person's satisfaction. Thus, we choose that arrangement: child A sits in the rear passenger seat, child B sits in the front passenger seat, and child C sits in the rear driver seat. □