Chapter 8 Section 2

MA1032 Data, Functions & Graphs

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Exercise

Let $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{1}{1-x}$, $f_5(x) = \frac{x-1}{x}$, and $f_6(x) = \frac{x}{x-1}$. Note that

$\circ$	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_1	f_4	f_3	f_6	f_5
f_3	f_3	f_5	f_1	f_6	f_2	f_4
f_4	f_4	f_6	f_2	f_5	f_1	f_3
f_5	f_5	f_3	f_6	f_1	f_4	f_2
f_6	f_6	f_4	f_5	f_2	f_3	f_1

Inverse

Definition

Suppose $Q = f(t)$ is a function with the property that each value of Q determines exactly one value of t. Then f has an inverse function, f^{-1} and

$$f^{-1}(Q) = t \text{ if and only if } Q = f(t).$$

If a function has an inverse, it is said to be invertible.

Properties

If $y = f(x)$ is an invertible function and $y = f^{-1}(x)$ is its inverse, then

- $f^{-1}(f(x)) = x$ for all values of x for which $f(x)$ is defined.
- $f(f^{-1}(x)) = x$ for all values of x for which $f^{-1}(x)$ is defined.

Example

Check that $g(x) = 1 - \frac{1}{x-1}$ and $f(x) = 1 + \frac{1}{1-x}$ are inverses of each other.

Finding an Inverse

Example

Find the inverse of $h(x) = \frac{\sqrt{x}}{\sqrt{x}+1}$.

Non-Invertible Functions

Example

Does $f(x) = x^2$ have an inverse?

Horizontal Line Test

- Domain of f^{-1} = Range of f
- Range of f^{-1} = Domain of f

Restricting the Domain

Could we make x^2 invertible?

Summary

- Definition
- Properties
- Finding an Inverse
- Domain & Range

Exercise

Let $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{1}{1-x}$, $f_5(x) = \frac{x-1}{x}$, and $f_6(x) = \frac{x}{x-1}$. Note that

$\circ$	f_1	f_2	f_3	f_4	f_5	f_6
f_1	f_1	f_2	f_3	f_4	f_5	f_6
f_2	f_2	f_1	f_4	f_3	f_6	f_5
f_3	f_3	f_5	f_1	f_6	f_2	f_4
f_4	f_4	f_6	f_2	f_5	f_1	f_3
f_5	f_5	f_3	f_6	f_1	f_4	f_2
f_6	f_6	f_4	f_5	f_2	f_3	f_1

Using the table above, find the following:

1. f_6^{-1}
2. $(f_3 \circ f_6)^{-1}$
3. F if $f_2 \circ f_5 \circ F = f_5$.