Chapter 9 Section 4

MA1032 Data, Functions & Graphs

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Rational Function

Definition

A rational function is a function, $r(x)$, which can be written as a ratio of two polynomial functions $p(x)$ and $q(x)$.

$$r(x) = \frac{p(x)}{q(x)}$$

Examples

- $f(x) = \frac{x-3}{x+2}$
- $g(x) = \frac{3x^2}{(x-1)(x-3)}$
- $h(x) = \frac{x^2+1}{x-2}$

Exercise #14

Bronze is an alloy, or mixture, of copper and tin. The alloy initially contains 3 kg copper and 9 kg tin. You add x kg of copper to this 12 kg of alloy. The concentration of copper in the alloy is a function of x:

$$f(x) = \text{concentration of copper} = \frac{\text{total amount of copper}}{\text{total amount of alloy}}.$$

a) Find a formula for f in terms of x, the amount of copper added.

b) Evaluate the following expressions and explain their significance for the alloy:

(i) $f(\frac{1}{2})$ (ii) $f(0)$ (iii) $f(-1)$ (iv) $f^{-1}(\frac{1}{2})$ (v) $f^{-1}(0)$

c) Graph $f(x)$ for $-5 \leq x \leq 5$, $-0.25 \leq y \leq 0.5$. Interpret the intercepts in the context of the alloy.

d) Graph $f(x)$ for $-3 \leq x \leq 100$, $0 \leq y \leq 1$. Describe the appearance for large x-values. Does the appearance agree with what you expect to happen when large amounts of copper are added to the alloy?

Long-run Behavior

Example

$$g(x) = \frac{3x^2}{x^2 - 4x + 3}$$

Practice

Exercise #1

Is $f(x) = \frac{x^2}{2} + \frac{1}{x}$ a rational function?

Exercise #6

Is $f(x) = \frac{9x-1}{4\sqrt{x}+7} + \frac{5x^3}{x^2-1}$ a rational function?

Exercise #10

Does $g(x) = \frac{(1-x)(2+3x)}{2x^2+1}$ have a horizontal asymptote? If so, what is it?

Summary

- Graphs of Rational Functions
- Formulas of Rational Functions
- Horizontal Asymptotes
- Long-run behavior