Tools 1

MA1032 Data, Functions & Graphs

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Definition

A linear equation in two variables, say the variables x and y, is an equation that can be written in the form $ax + by = c$ where a, b, and c are constants and a and b are not both zero.

Example

Determine if the following equations are linear.

1. $3x - (2 - 4y) = x - y + 1$
2. $\frac{x+2}{3} - y = \frac{y}{5}$
3. $x^2 - (x - 3)^2 = 3y$

Solving Exactly vs. Solving Approximately

Example

Solution: $\frac{3\pi}{\sqrt{2}}$ or 6.66432440724

Each has its benefits.

Definition

A system of equations is a group of equations.

Definition

The simultaneous solution to a system of equations is a solution that satisfies all of the equations in the system.

Example

Show that the coordinate $(4, -1)$ is the simultaneous solution to the following system of equations.

$$\begin{cases} x + y &= 3 \\ x - y &= 5 \end{cases}$$

Example

Is the coordinate $(1, 2)$ the simultaneous solution to the following system of equations?

$$\begin{cases} 3x - 2y = 6 \\ y = 2x - 5 \end{cases}$$

Methods for solving systems of equations.

1. Substitution.

   Solve for one variable in an equation and then plug it into the others.

2. Elimination.

   Multiply one equation by a convenient constant and then add the equation to another equation.

Example

Solve the following systems of equations.

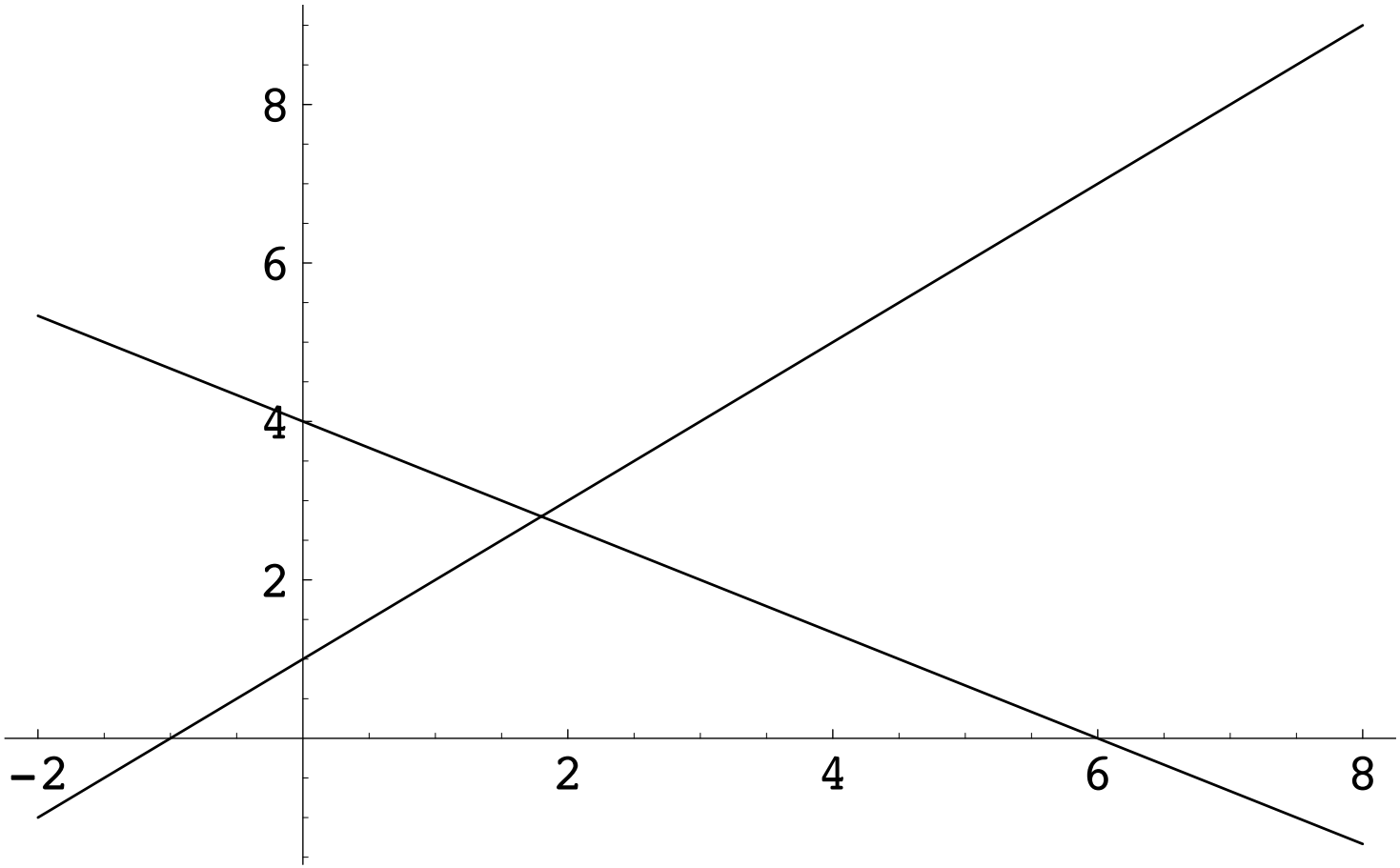
1. $\begin{cases} 2x - y = 10 \\ x + 2y = 15 \end{cases}$
2. $\begin{cases} x = 7y - 9 \\ 4x - 15y = 26 \end{cases}$
3. $\begin{cases} 3x - y = 17 \\ -2x - 3y = -4 \end{cases}$
4. $\begin{cases} 2x + 3y = 7 \\ y = -\frac{3}{5}x + 6 \end{cases}$

Application.

Finding the intersection of two lines.

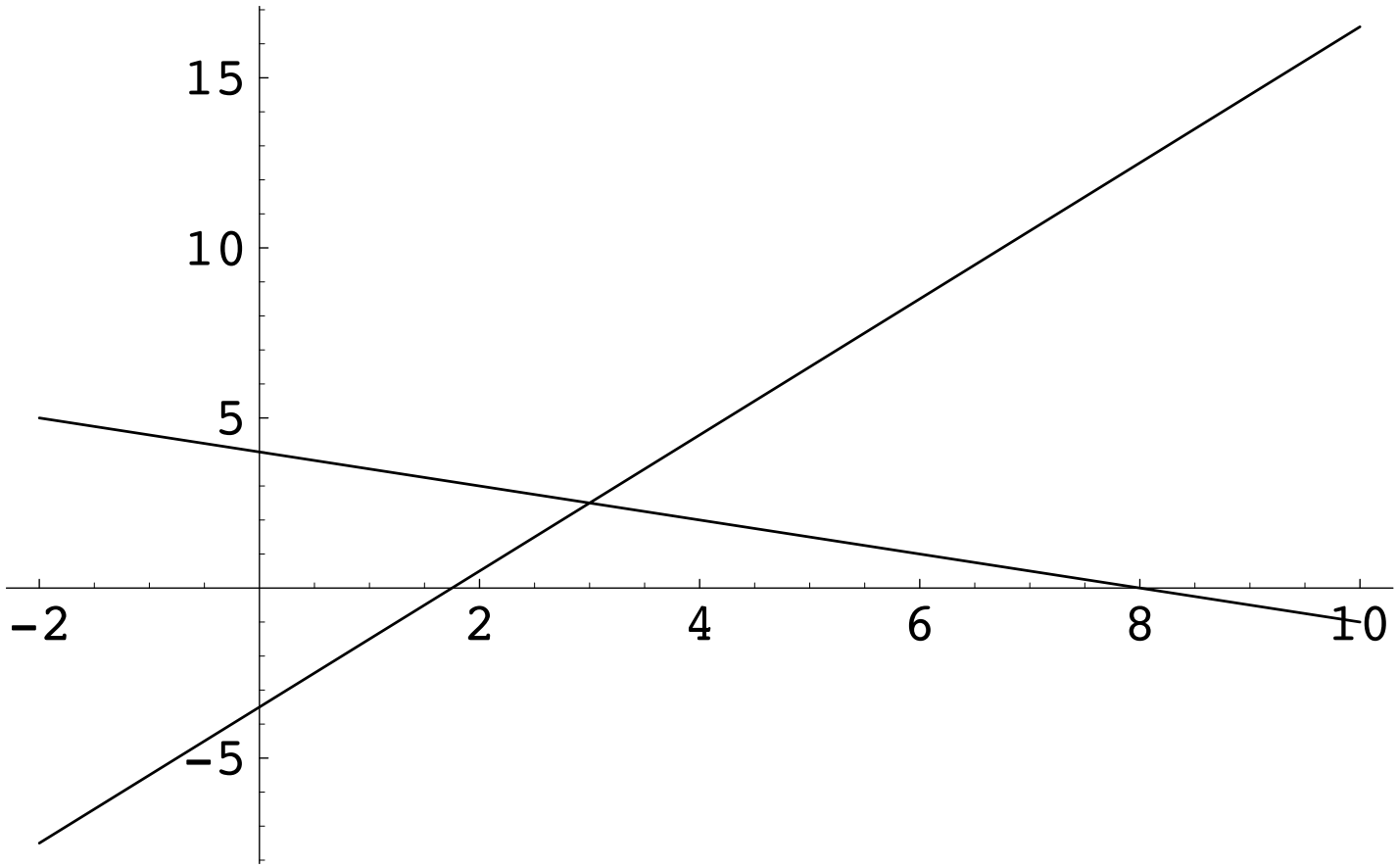
Example

Find the intersection of the lines $y = x + 1$ and $2x + 3y = 12$.



Example

Where do the lines $y = 2x - 3.5$ and $y = -\frac{1}{2}x + 4$ intersect?



Summary.

1. Linear Equations
2. Exact vs. Approximate Solutions
3. Systems of Equations
4. Substitution & Elimination