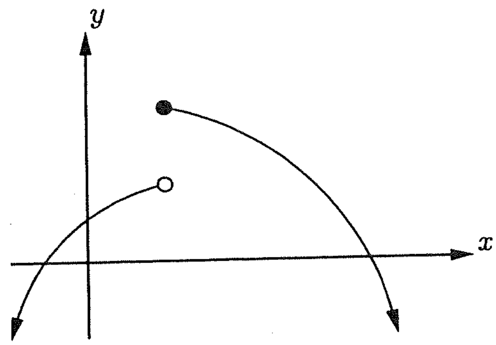
MA1032
Midterm Exam
Fall 2006

Name: Answer Key

Non-calculator section: You may not use your calculator on this section. You must show enough work to justify all answers. Once you have handed in this section you may not have it back.

1. Consider the following graph.



(a) Is y a function of x? Explain. [4] 6

(yes since for each x value, there is only one y value assigned according to the x value.

(3) Explanation

(b) Is x a function of y? Explain. [4] 6

(No. For some y values, there is more than one x value assigned according to each y value.

Explanation

2. Find the average rate of change of $f(x) = 3x^2 + 1$ between $x = 1$ and $x = 2$.

$$f(1) = 3 \cdot 1^2 + 1 = 4$$

$$f(2) = 3 \cdot 2^2 + 1 = 13$$

$$\frac{f(2) - f(1)}{2 - 1} = \frac{13 - 4}{1} = 9$$

9 [4] 6

3. There is a linear relationship between the number of cigarettes a person smokes and the probability of contracting lung cancer. The probability of a male of average health who smokes 10 cigarettes per day of getting lung cancer in the next 10 years is 0.02. The probability if he smokes 20 cigarettes per day is 0.04.

(a) Find a formula for $P(x)$, the probability that a male who smokes x cigarettes per day will contract lung cancer.

$P(x) = b + mx$

(10, 0.02) and (20, 0.04) should be satisfy this function.

$\left(m = \dfrac{0.04 - 0.02}{20 - 10} = \dfrac{0.02}{10} = 0.002 \right.$

$0.04 = (0.002)(20) + b$

$0.04 = \frac{2}{1000} \cdot 20 + b$

$0.04 = \frac{4}{100} + b$

$0.04 = 0.04 + b$

$0 = b$

$P(x) =$ $0.002x$ [redacted] 6

(b) Evaluate $P(0)$. What does this mean in practical terms? [redacted] 6

$P(0) = 0.02 \cdot 0 = 0$

P(0) means the probability that a male gets lung cancer in the next 10 years is 0 if he does not smoke.

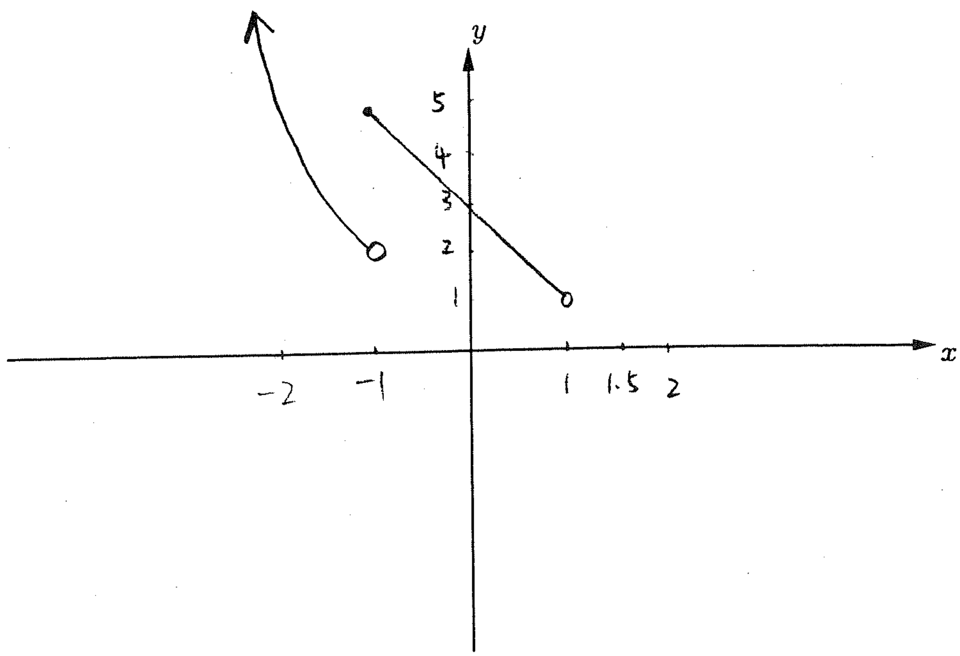
(c) Interpret the slope of the function P in practical terms. Be specific and include units. [redacted] 6

The slope of the function P is 0.002 probability/cigarette.

It means that the probability will increase by 0.002 if a male smokes an additional cigarette.

4. (a) Draw the graph of the following piecewise function. [redacted] 8

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < -1 \\ -2x + 3 & \text{if } -1 \leq x < 1 \end{cases}$$



(b) Evaluate $f(-2), f(-1)$.

$f(-2) = (-2)^2 + 1 = 4 + 1 = 5$

$f(-1) = -2 \cdot (-1) + 3 = 2 + 3 = 5$

$f(-2) =$ 5 [2]

$f(-1) =$ 5 [2]

5. Suppose f is a function of x with a formula:

$$f(x) = \frac{\sqrt{3x-4}}{x^2-25}.$$

(a) What is the domain of this function?

$x^2 - 25 = (x-5)(x+5) \neq 0 \quad x \neq 5 \text{ or } x \neq -5$

$3x - 4 \geq 0 \quad x \geq \frac{4}{3}$

Domain: $\{x \mid x \geq \frac{4}{3}, x \neq 5\}$

[4] 6

6. Solve the following equation.

$$-2x^2 + 4x + 1 = 0$$

$a = -2 \quad b = 4 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4\cdot(-2)\cdot 1}}{2\cdot(-2)}$$

$$= \frac{-4 \pm \sqrt{16+8}}{-4}$$

$$= 1 \pm \frac{2\sqrt{6}}{-4}$$

$1 - \sqrt{6}$

$1 - \frac{\sqrt{6}}{2}$ or $1 + \frac{\sqrt{6}}{2}$

[4] 6

7. Suppose A is a function of t with a formula:

$$A = g(t) = -\frac{2}{t+4} - 1.$$

Find a formula for the inverse function, $g^{-1}(A)$.

$$(t+4)\left(A = \frac{-2}{t+4} - 1\right)$$

$$A(t+4) = -2 - (t+4)$$

$$At + 4A = -2 - t - 4$$

$$At + t = -6 - 4A$$

$$t(A+1) = -6 - 4A$$

$$t = \frac{-6-4A}{A+1}$$

$g^{-1}(A) =$ $\frac{-6-4A}{A+1}$, $A \neq -1$ [6]

8. Solve the following for x. Give exact answers.

(a) $\log(x + 19) = 1.8$

$$x + 19 = 10^{1.8}$$

$$x = 10^{1.8-19}$$

$$10^{1.8} = x + 19$$

$$x = 10^{1.8} - 19$$

$x =$ $10^{1.8} - 19$ [6]

(b) $e^{5x} = 3.2$

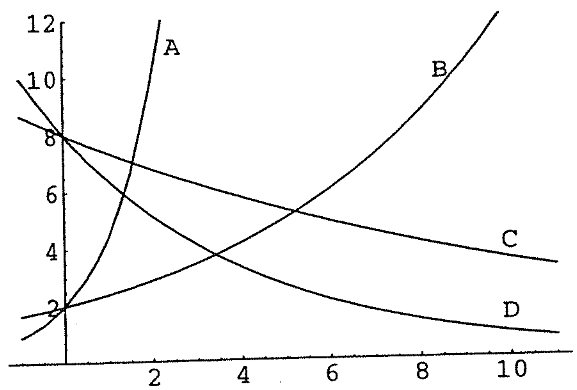
$$\ln e^{5x} = \ln 3.2$$

$$5x = \ln 3.2$$

$$x = \frac{\ln 3.2}{5}$$

$x =$ $\frac{\ln(3.2)}{5}$ [6]

9. Match each graph to a function.



$f(x) = 8(0.92)^x$ C

$g(x) = 2(1.2)^x$ B

$h(x) = 2e^{0.8x}$ A

$j(x) = 8(0.80)^x$ D [8]

10. Write an expression for the balance, but **do not evaluate.**

(a) \$300 deposited at 4% annual interest compounded annually for 5 years

$300(1+0.04)^5$ [4]

(b) \$400 deposited at 5% annual interest compounded monthly for 3 years

$400\left(1+\frac{0.05}{12}\right)^{12\cdot 3}$ [4]

(c) \$500 deposited at 3% annual interest compounded continuously for 4 years

$500e^{(0.03)(4)}$ [4]

1. The formula $D = 5e^{-0.4h}$ gives the amount of drug, D, in milligrams, that remains in a patient's bloodstream h hours after the drug is injected.

(a) How many milligrams of the drug are injected?

5 mg [4]

(b) How long will it take for the amount of drug remaining in the bloodstream to reach 2 milligrams? Give your answer to four decimal places.

$$2 = 5e^{-0.4h}$$

$$\ln \frac{2}{5} = \ln e^{-0.4h}$$

$$\ln \frac{2}{5} = -0.4h$$

$$2.2907 \approx \frac{\ln\left(\frac{2}{5}\right)}{-0.4} = h$$

2.2907 hrs [6]

2. (a) Convert $Q = 30(1.12)^t$ to the form $Q = ae^{kt}$. Give any decimals to four places.

$1.12 = e^k$

$\ln 1.12 = \ln e^k$

$0.1133 \approx \ln 1.12 = k$

$Q =$ $30e^{0.1133t}$ [6]

(b) Assuming t is in years, use the above formulas to state the annual percent growth rate and the continuous percent growth rate.

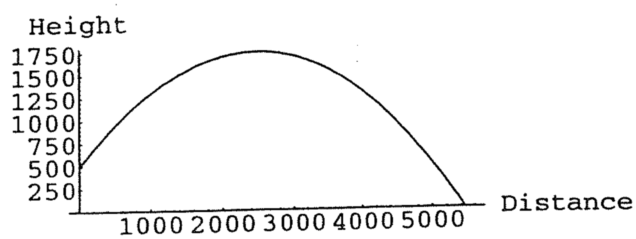
Annual rate: 12% [3]

Continuous rate: 11.33% [3]

3. 4. A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. In physics, it is established that the height h of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(400)^2} + x + 500$$

where x is the horizontal distance of the projectile from the base of the cliff.



(a) Find the maximum height of the projectile.

$a = -\frac{32}{(400)^2}$ $\quad b = 1 \quad c = 500$

$x = -\frac{b}{2a} = -\frac{1}{2\cdot\left(\frac{-32}{400^2}\right)} = 25$

$h(25) = \frac{-32}{400^2} \cdot 25^2 + 25 + 500$

$= 512.5$

512.5 feet [4] 6

(b) How far from the base of the cliff will the projectile strike the water?

$0 = \frac{-32x^2}{400^2} + x + 500$

$x = \frac{-1 \pm \sqrt{1 - 4\cdot\left(\frac{-32}{400^2}\right)\cdot 500}}{2\cdot\left(\frac{-32}{400^2}\right)}$

$= \frac{-1 \pm \sqrt{41}}{-0.04} = \frac{-1 \pm 6.40312}{-0.04}$

185.078 feet [4] 6

4. Three students used 9-volt batteries to charge small capacitors. Then each recorded the voltage of the capacitor over the next 30 seconds. Each student found a model to fit the recorded data. The model the first student gave was $V(t) = 8.73(1 - 0.118)^t$, with V measured in volts and t in seconds.

(a) Interpret in practical terms the numbers 8.73 and $-.118$ in this equation. [6]

The initial voltage is 8.73 Volts

The voltage decreases by 11.8 % every second

(b) The second student gave the model for her transistor as $V(t) = 8.91e^{-.124t}$. Which of the two transistors was losing voltages faster? Explain how you know. [6]

1st transistor

$r = 0.118$

$= 11.8\%$ decrease

2nd transistor

$b = e^{-0.124} = 0.88338$

$b = 1 - r \Rightarrow r = 1 - b = 1 - 0.8838$

$= 0.1162$

$= 11.62\%$ decrease

The first transistor loses voltage at a rate of 11.8 % every second. The 2nd transistor loses voltage at a rate of 11.62 % per second. Therefore the first transistor loses voltage faster.

(c) The third student had some difficulty making measurements and recorded only two observations. At 10 seconds the voltage was 2.88 volts, and it had dropped to 0.31 volts at 30 seconds. Find an exponential model for this data.

$(10, 2.88)$ $(30, 0.31)$

$V = ab^t$

$$\frac{0.31 = \cancel{a}b^{30}}{2.88 = \cancel{a}b^{10}}$$

$$\left(\frac{0.31}{2.88}\right)^{\frac{1}{20}} = \left(b^{20}\right)^{\frac{1}{20}}$$

$b = 0.8945$

$V = a(0.8945)^t$

$0.31 = a(0.8945)^{30}$

$8.7892 = a$

$V(t) = 8.7892(0.8945)^t$ [6]