Quiz 2 - 2.6, 5.5, Tools 3, 3.1, & 3.2

Solutions

October 10, 2006

Show all work for full credit. If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator when an algebraic method is available, you will not receive full credit.

1. Find the formula for the function y = f(t) which is a parabola with vertex (3,0) and y-intercept -4. Solution.

$$f(t) = a(t-3)^2$$

To solve for a, we plug in the point (0, -4).

$$-4 = a(0-3)^2$$
$$a = \frac{-4}{9}$$

Thus, $y = f(t) = \frac{-4}{9}(t-3)^2$.

- 2. Put $f(x) = x^2 + 6x 2$ in vertex form. Solution. The vertex is (-3, -11). Thus, $f(x) = a(x+3)^2 - 11$. Factoring, we get $f(x) = a(x^2 + 6x + 9) - 11$. If a = 1, then we get the original formula. Therefore, $f(x) = (x+3)^2 - 11$ is in vertex form.
- 3. Kevin buys a new CD player for \$250.00, and finds two years later when he wants to sell it that it is only worth \$100.00. Find a formula for the value of the CD player if the value decreases:

Solution. The two points given in the problem are (0, 250) and (2, 100).

(a) linearly.

Using the two points, we find the slope to be -75. Noticing that the first point is the initial value, or vertical-intercept, we get y = f(x) = -75x + 250 as a linear formula that describes the value of the CD player.

(b) exponentially.

Using the two points, we find two equations in exponential form.

$$250 = ab^0$$
 and $100 = ab^2$

The first equation simplifies to yield a = 250. Substituting this value of a into the other equation yields $b = \left(\frac{2}{5}\right)^{\frac{1}{2}}$. Therefore, we get $y = g(x) = 250\left(\frac{2}{5}\right)^{\frac{x}{2}}$ is an exponential function describing the value of Kevin's CD player.

The following questions are related to the Test 1 material. Your scores along with those on future quizzes will be used to determine if you have mastered the Test 1 material.

- 1. Suppose $f(x) = x^2 5$.
 - (a) Find the average rate of change of the function f between x = 3 and x = 6. Solution.

$$\frac{f(6) - f(3)}{6 - 3} = \frac{31 - 4}{3} = 9$$

(b) Find the value of c making the average rate of change between x = 3 and x = c equal to 10.

Solution.

$$\frac{f(c) - f(3)}{c - 3} = \frac{c^2 - 5 - (3^2 - 5)}{c - 3} = \frac{c^2 - 9}{c - 3} = 10$$

The quadratic formula is invoked to solve for c.

$$c^{2} - 10c + 21 = 0$$
$$c = \frac{10 \pm \sqrt{100 - 4(21)}}{2} = 7,3$$

The value of c that makes sense in the context of the problem in c = 7.

2. Suppose that your uncle Gerald has just opened a store that sells television sets. Let P(x) represent the profit that uncle Gerald makes by selling x television sets per week. Suppose that uncle Gerald currently sells k television sets per week. Using complete sentences, state in everyday real-world terms, involving profit and television sets, what each of the following means. Do not use symbols x, k, or P directly in your answers. Thus, for example, if you had to refer to P(k), you would instead write "the profit uncle Gerald is making at his current level of sales."

(a) P(k) + 1000

Solution. This formula represents \$1000 more than the profit uncle Gerald is making at his current level of sales. $\hfill \Box$

(b) P(k+5)

Solution. This formula represents the profit uncle Gerald would make if he sold five more television sets per week over his current level of sales. \Box

(c) P(2k)

Solution. This formula represents the profit uncle Gerald would make if he sold twice as many television sets per week as his current level of sales. $\hfill \Box$