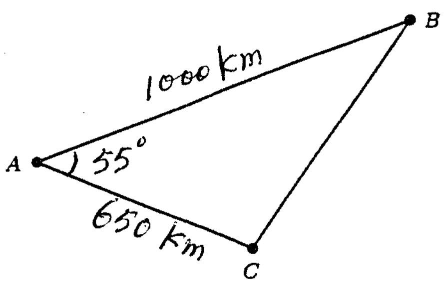
MA1032

Final Exam Review
Key
(Sections 7.1 - 9.2)

1. The boundaries of the Bermuda Triangle are not universally agreed upon, but one version places one corner of the triangle on the southern US coast (Point A in the figure), another corner in the Bermuda Islands (point B), and the third corner in the Greater Antilles (point C). Assume that the distance between A and B is 1000 km, and the angle at point A is 55°. Traveling at 25 km/hr, a cruise ship takes 26 hours to cross from point A to point C. If the ship survives this portion of the trip, what is the distance it must travel from point C to point B?



$|AC| = 26 \cdot 25 = 650 \text{ km}$

$|BC|^2 = 1000^2 + 650^2 - 2 \cdot 1000 \cdot 650 \cdot \cos 55^\circ$
$= 676851$

$|BC| = \sqrt{676851} = 822.71 \text{ km}$

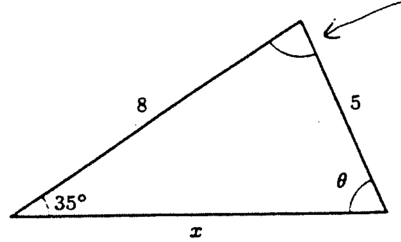
2. Let θ and φ be the two angles shown below and suppose 0 < θ < 90° and 90° < φ < 180°.

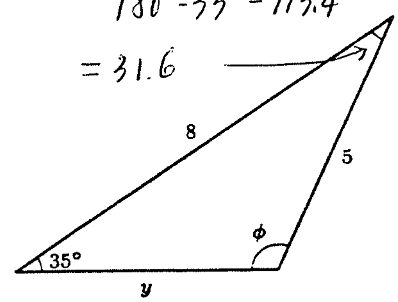
$180^\circ - 35^\circ - 66.6^\circ = 78.4^\circ$

$180^\circ - 35^\circ - 113.4^\circ = 31.6$

Law of Sines

SSA





a. Find θ and φ.

$\dfrac{\sin\theta}{8} = \dfrac{\sin 35^\circ}{5}$ $\quad \sin\theta = \dfrac{8\sin 35^\circ}{5} = 0.918$ $\quad \dfrac{\sin\phi}{8} = \dfrac{\sin 35^\circ}{5}$ $\quad \sin\phi = \dfrac{8\sin 35^\circ}{5}$

$\theta = \sin^{-1}(0.918) = 66.6^\circ$ $\quad \sin^{-1}(0.918) = 66.6^\circ$

$\phi = 180^\circ - 66.6^\circ = 113.4^\circ$

b. Find x and y.

$\dfrac{\sin 78.4^\circ}{x} = \dfrac{\sin 35^\circ}{5}$ $\quad x = \dfrac{5\sin 78.4^\circ}{\sin 35^\circ} = 8.54$

$\dfrac{\sin 31.6^\circ}{y} = \dfrac{\sin 35^\circ}{5}$ $\quad y = \dfrac{5\sin 31.6}{\sin 35^\circ} = 4.57$

3. Establish the identity:

$$\frac{\cos 2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$$

$$\frac{\cos 2\theta}{\cos\theta - \sin\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta - \sin\theta}$$

$$= \cos\theta + \sin\theta$$

4. Complete the following table, using exact values where posssible.

$\sin\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

θ (radians)	$\sin^2\theta$	$\cos^2\theta$	$\sin 2\theta$	$\cos 2\theta$
0	0	1	0	1
π/4	½	½	1 $\sin\frac{\pi}{2} = 1$	0 $\cos\frac{\pi}{2} = 0$
π	0	1	0	1

5. Complete the following table for $m(x) = f(g(x))$ and $n(x) = f(x) + g(x)$.

x	0	1	2	3	4	5
$f(x)$	2	0	5	1	3	4
$g(x)$	1	2	4	0	5	3
$m(x)$	0	5	3	2	4	1
$n(x)$	3	2	9	1	8	7

$f(g(0)) = f(1) = 2$

$f(g(1)) = f(2) = 5$

6. Find a formula for $g(f(x))$ when $f(x) = 1/x$ ($x \neq 0$) and $g(x) = x^2 - 1$. ($x \in \mathbb{R}$)

$$g(f(x)) = \left(\frac{1}{x}\right)^2 - 1 = \frac{1}{x^2} - 1 = \frac{1-x^2}{x^2}$$

domain $\{x \mid x \neq 0\}$

7. Find a formula for $f^{-1}(x)$ if $f(x) = \dfrac{3x+1}{x-2}$. $f\ \{x \mid x \neq 2\}$ $f^{-1}\ \{x \mid x \neq 3\}$ $\{y \mid y \neq 3\}$ $\{y \mid y \neq 2\}$

$x = \dfrac{3y+1}{y-2}$

$x(y-2) = 3y+1$

$xy - 3y = 1 + 2x$

$y(x-3) = 1 + 2x$

$y = \dfrac{1+2x}{x-3}$

$$y = \frac{3x+1}{x-2}$$

$$3x + 1 = y(x-2)$$

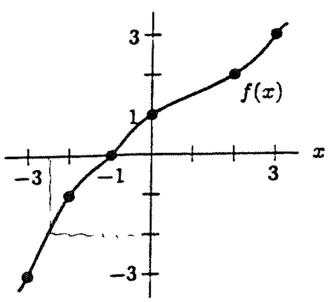
$$3x + 1 = xy - 2y$$

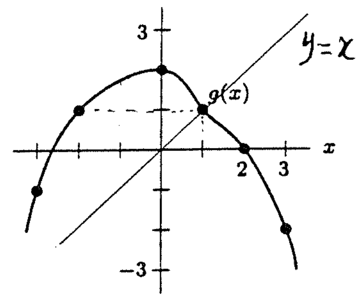
$$(3-y)x = -2y - 1$$

$$x = \frac{-2y-1}{3-y} = \frac{1+2y}{y-3}$$

$$f^{-1}(x) = \frac{1+2x}{x-3}$$

8. Given the graphs of f and g below, answer the following.





a. Evaluate $f(g(3)) = f(-2) = -1$

b. Evaluate $f^{-1}(0) = -1$, because $f(-1) = 0$ what input makes output 0

c. Estimate $f^{-1}(-2) = -2.5$ because $f(-2.5) = -2$

What input gives output of -2

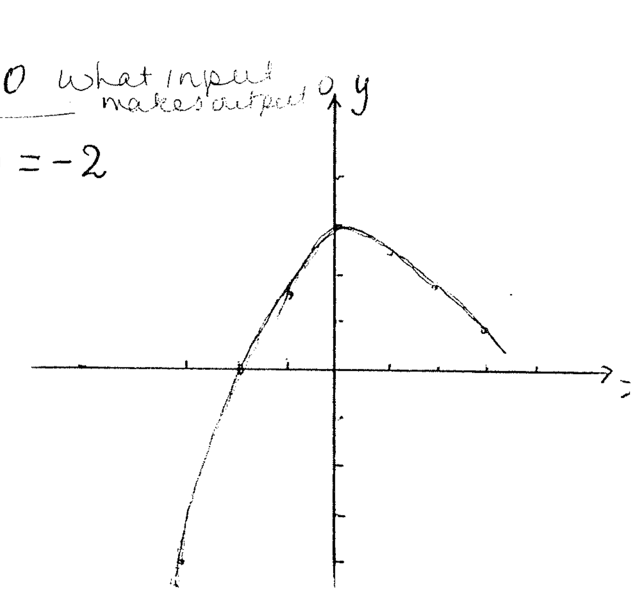
d. Solve $g(x) = 1$ $x = 1$ and -2

e. Solve $f(x) = x$ $x = 1$

Remember domain is ∩ of 2 domains

f. Graph $f(x) + g(x)$

x	−3	−2	−1	0	1	2	3
$f(x)+g(x)$	−4	0	1.8	3	2.5	2	1



9. Find a simplified formula for the following functions. Let $f(x) = x + 2$, $g(x) = x^3$, and $h(x) = \sqrt{x - 1}$.

$x \in \mathbb{R}$ $x \in \mathbb{R}$

$x - 1 \geq 0$
$x \geq 1$

domain

a. $2f(x) - g(x) = 2(x+2) - x^3 = 2x + 4 - x^3$ $x \in \mathbb{R}$

b. $\dfrac{f(x)}{g(h(x))} = \dfrac{x+2}{(\sqrt{x-1})^3} = \dfrac{x+2}{|x-1| \cdot \sqrt{x-1}}$ or $\dfrac{x+2}{(x-1)^{3/2}}$ $x \geq 1$

10. Are the following functions power functions? If so, write them in the form $y = kx^p$.

a. $y - 4 = (x + 2)(x - 2)$

$y - 4 = x^2 - 4$
$y = x^2$ so yes

b. $y = (x - 3)^2$ No

11. Find a power function through the point (1, 4) and (4, 7).

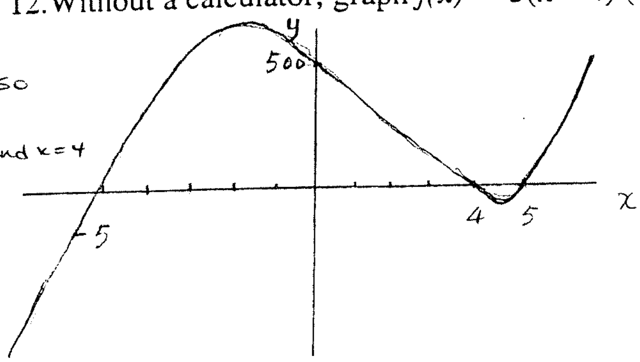
$\boxed{y = 4x^{0.4037}}$

$y = kx^p$ $\begin{cases} 4 = k \cdot 1^p = k \\ 7 = k \cdot 4^p \end{cases}$ $k = 4$

$7 = 4 \cdot 4^p$ $4^p = \dfrac{7}{4}$ $p \ln 4 = \ln \dfrac{7}{4}$ $p = \dfrac{\ln \frac{7}{4}}{\ln 4} = 0.4037$

12. Without a calculator, graph $f(x) = 5(x - 4)(x^2 - 25)$. Label all x and y–intercepts

$f(0) = 500$ so
$f(x) > 0$
b/w $x = -5$ and $x = 4$



zeros are : $x = 4$, $x = 5$, $x = -5$
all odd so cross
$f(0) = 5(-4)(-25) = 500$.
long-run behavior : $y = 5x^3$
$x \to \infty$, $y \to \infty$
$x \to -\infty$, $y \to -\infty$

13. Find the zeros of the function $y = x^3 - 2x - 8x$.

$x^3 - 2x - 8x = 0$, $x^3 - 10x = 0$ $x(x^2 - 10) = 0$

$x(x - \sqrt{10})(x + \sqrt{10}) = 0$ $x = 0, \sqrt{10}, -\sqrt{10}$.

The zeros are $0, \sqrt{10}, -\sqrt{10}$.

14. For the following polynomials, state the degree, the number of terms, and describe the long–run behavior.

a. $y = 1 - 2x^4 + x^3$

as $x \to \pm\infty$, $y \to -\infty$

degree: 4 # of terms: 3 long-run behavior: $y = x^4$

b. $y = (x + 4)(2x - 3)(5 - x)$

as $x \to +\infty$, $y \to -\infty$, as $x \to -\infty$, $y \to \infty$

degree: 3 # of terms: 4. long-run behavior: $y = -2x^3$

15. Find a possible formula for a polynomial f with the following properties: f is third degree with $f(-3) = 0$, $f(1) = 0$, $f(4) = 0$, and $f(2) = 5$.

$f(x) = a(x+3)(x-1)(x-4)$, $5 = a(2+3)(2-1)(2-4)$

$a = \dfrac{5}{-10} = -\dfrac{5}{10}$ $\quad f(x) = -\dfrac{5}{10}(x+3)(x-1)(x-4)$

$= -\frac{1}{2}(x+3)(x-1)(x-4)$

16. Find the horizontal asymptote, if it exists of

$\dfrac{2+x-3x^2}{2x^2+1} \leftarrow g(x) = \dfrac{(1-x)(2+3x)}{2x^2+1} \approx \dfrac{-x \cdot 3x}{2x^2} = -\dfrac{3}{2}$

The horizontal asymptote is $y = -\frac{3}{2}$

17. An alcohol solution consists of 5 gallons of pure water and x gallons of alcohol, $x > 0$. Let $f(x)$ be the ratio of the volume of alcohol to the total volume of liquid. (Note that $f(x)$ is the concentration of the alcohol in the solution).

a. Find a possible formula for $f(x)$.

$f(x) = \dfrac{\text{Amount of Alcohol}}{\text{Amount of Liquid}} = \dfrac{x}{x+5}$

b. Evaluate and interpret $f(7)$ in the context of the mixture.

$f(7) = \dfrac{7}{7+5} = \dfrac{7}{12} \approx 58.33\%$

58.33% is the concentration of alcohol in a solution consisting of 5 gallons of water and 7 gallons of alco

c. What is the zero of f: Interpret your result in the context of the mixture.

$f(x) = 0 \Rightarrow \dfrac{x}{x+5} = 0 \Rightarrow x = 0$.

The concentration of alcohol is 0% when there is no alcohol in the solution.

d. Find an equation for the horizontal asymptote of f. Explain its significance in the context of the mixture.

$\dfrac{x}{x+5} \approx \dfrac{x}{x} = 1$, $y = 1$ is the horizontal asymptote of f.

This means that as the amount of alcohol added, x, grows large, the concentration of alcohol in the solution approaches 1c

18. What are the x–intercepts, y–intercepts, and horizontal and vertical asymptotes (if any?

$$y = \frac{(x-4)}{(x^2-9)}$$

$x=4,\ y=0 \Rightarrow x$ - intercept is $(4,0)$

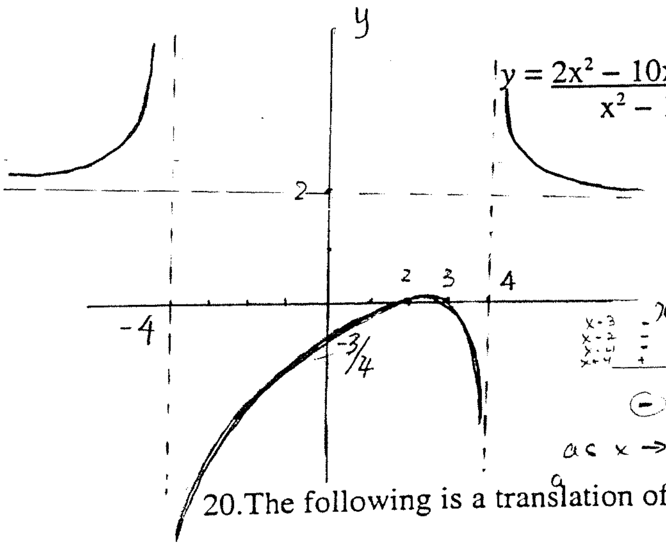
$x=0,\ y=4/9 \Rightarrow y$ - intercept is $(0, 4/9)$

$x \to \infty,\ y \to 0 \Rightarrow y=0$ is the horizontal asymptotes

$x^2-9=0 \Rightarrow x=\pm 3 \Rightarrow x=\pm 3$ are the vertical asymptotes

19. Graph the function

$$y = \frac{2x^2-10x+12}{x^2-16} = \frac{2(x^2-5x+6)}{(x-4)(x+4)} = \frac{2(x-3)(x-2)}{(x-4)(x+4)}$$



x-intercepts : $(3,0)$, $(2,0)$

y-intercepts : $(0, -3/4)$

horizontal asy. : $y=2$

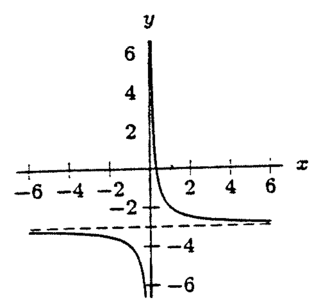
Vertical asy. : $x=4$ and $x=-4$

$x-3$, $x-2$, $x-4$, $x+4$; 3.5 can't do; $-$ 2 $+$ 3 $-$

as $x \to -4^-$ $f(-5)=1.33$, $f(-4.5) = 18.6$ so $f(x) \to \infty$

as $x \to 4^+$ $f(5)=1.33$, $f(4.5)=18.6$ so $f(x) \to \infty$

20. The following is a translation of $y = \frac{1}{x}$



$y = \frac{1}{x}$ shifted dow 3 units

a. Find a possible formula for the graph.

$y = \frac{1}{x} - 3 = \frac{1-3x}{x}$

b. Write the formula from part (a) as the ratio of two linear polynomials.

$y = \frac{1}{x} - 3 = \frac{1}{x} - \frac{3x}{x} = \frac{1-3x}{x} = \frac{-3x+1}{x}$

c. Find the coordinates of the intercepts of the graph.

$x \neq 0$ so no y-int

The graph has no y-intercept since $x=0$ is not in the domain of the function.

$\frac{-3x+1}{x} = 0 \Rightarrow -3x+1=0 \Rightarrow x = 1/3$

The x-intercept is $(1/3, 0)$