MA1032 – Common Midterm Review

Solutions

October 2006

1. Find the zeros of $g(t) = 6t^2 - 5t - 1$.
   Solution.
   $$0 = 6t^2 - 5t - 1$$
   $$0 = (6t+1)(t-1)$$
   $$\begin{array}{ccc} 6t + 1 = 0 & \text{or} & t - 1 = 0 \\ t = \frac{-1}{6} & & t = 1 \end{array}$$
   □

2. Find two points on the graph of $y = K(x) = 6 - x^2$ whose y-coordinates are -3.
   Solution.
   $$6 - x^2 = -3$$
   $$9 - x^2$$
   $$x = \pm 3$$
   So the points are $(3, -3)$ and $(-3, -3)$. □

3. A ball is thrown into the air. The height of the ball above the ground is given by $S(t) = -16t^2 + 64t + 3$. Determine when the all hits the ground.
   Solution.
   $$-16t^2 + 64t + 3 = 0$$
   $$t = \frac{-62 \pm \sqrt{64^2 - 4(-16)(3)}}{-32} \approx 4.05 \text{ and} - 0.05$$
   Since $t > 0$, the ball hits the ground at 4.05 seconds. □

4. Simplify and write each expression so that all exponents are positive.

   (a) $\frac{x^5 y^{-2}}{x^3 y}$
   Solution. $\frac{x^2}{y^3}$ □

(b) $\left(\frac{x^3}{3y^{-1}}\right)^2$
Solution. $\frac{x^6y^2}{9}$ ☐

(c) $\frac{(-2)^3x^4(yz)^2}{(3)^{-2}xy^3z}$
Solution. $\frac{72x^3z}{y}$ ☐

(d) $\left(\frac{5x^{-2}}{6y^{-2}}\right)^{-3}$
Solution. $\frac{216x^6}{125y^6}$ ☐

(e) $5^{-3/4}$
Solution. $\frac{1}{5^{3/4}}$ ☐

5. The following formula gives the quantity of carbon-14 in grams over 1000 years. Describe in words how the quantity is changing over time and what the initial size initially was.

$$Q = f(t) = 200(0.886)^t$$

Solution. Q is an exponential decay function with an initial quantity of carbon-14 of 200 grams and it is decaying at a rate of 11.4% per year. ☐

6. A population has a size of 600 in 1990 ($t = 0$).

   (a) If the population grows by 50 people per year, find a formula for the population, P, at time t.
   Solution. $P = f(t) = 600 + 50t$ ☐

   (b) If the population grows by 8% per year, find a formula for the population, P at time t.
   Solution. $P = f(t) = 600(1.08)^t$ ☐

7. The table below shows v, the dollar value of a share of a certain stock, as a function of t, the time (in weeks) since the initial offering of the stock. Show that this data could be described by an exponential function.
Solution. The ratio of consecutive y-values is constant–it is always 0.7578, so v could describe an exponential function. ☐

8. The population of Seattle has been growing at a rate of 6% per year. If the population was 100,000 in 1960, what was the projected population for 1998?
Solution. $P = f(t) = 100,000(1.05)^t$. If $t = 0$ in 1960, then $t = 38$ in 1998. So $f(38) = 100,000(1.05)^{38} = 638,548$. ☐

9. The graph of $P(t)$, an exponential function, follows:

   (a) Find a formula of $P(t)$.
   Solution. $P = f(t) = 625(0.8)^t$ ☐

(b) Suppose $P(t)$ represents a city's population, in thousands, t years after 1980. Evaluate the expression $P(10) - P(5)$. what does this expression represent in the context of the city's population?

Solution. $P(10) - P(5) \approx -138$ thousand. This represents the decrease in population from 1985 to 1990. The population decreased by 138 thousand people. □

10. Let $P(t) = 1000e^{0.041t}$ give the size of a population of animals in year t.

(a) Briefly describe this population in words. Be specific.
Solution. The population has an initial size of 1000 and grows at a continuous rate of 4.1%. □

(b) Evaluate $P(15)$. Explain what this tells you about the population.
Solution. $P(15) \approx 1850$. After 15 years the population size is 1850. □

(c) Solve $P(t) = 5000$ for t. What do your solution(s) tell you about the population?
Solution. $t = 39.25$. It took $39\frac{1}{4}$ years for the population to increase to 5 times its original size. □

11. A bank pays interest at the nominal rate of 4.2% per year. What is the effective annual yield if compounding is:

(a) annual
Solution. $(1 + \frac{0.42}{1})^{(1)(1)} = 1.042 so r=0.042 or 4.2\%$ □

(b) monthly
Solution. $(1 + \frac{0.042}{12})^{12(1)} = 1.0428$ so $r = 0.0428$ or 4.28% □

(c) continuous
Solution. $e^{0.042(1)} = 1.04289$ so $r = 0.04289$ or 4.29% □

12. Evaluate without a calculator: $\log(1/\sqrt{10}) + \ln(e^{2x}) + e^{\ln(5-3x)}$.
Solution. $\frac{9}{2} - x$ □

13. Solve for x: $5(3^x) - 4 = 12$
Solution. $x = 1.059$ □

14. Let $\log A = 3$ and $\log B = 2$. Evaluate the following expressions, if possible.

(a) $\log(AB) = 5$

(b) $\log(A^3B^2) = 13$

(c) $\log(A/B) = 1$

(d) $\frac{\log A}{\log B} = \frac{3}{2}$

15. A population grows from 1000 to 1700 in 5 years. Find the following.

(a) annual growth rate

Solution.

$$P = f(t) = 1000b^t$$
$$1700 = 1000b^5$$
$$1.7 = b^5$$
$$1.112 = b$$

So $r = 0.112$ or 11.2%. □

(b) continuous growth rate

Solution.

$$P = f(t) = 1000e^{kt}$$
$$1700 = 1000e^{5k}$$
$$1.7 = e^{5k}$$
$$\frac{\ln 1.7}{5} = k$$

So $k = 0.1061$ or 10.61%. □

(c) doubling time

Solution.

$$t = \frac{\ln 2}{\ln(1.112)}$$

So $t = 6.53$ years. □

16. The half life of an element is 15 hours. If you have 10 grams to start with, how many hours will it take until you have only 3 grams?

Solution. Find b:

$$5 = 10b^{15}$$
$$b = 0.9548.$$

Then

$$Q = f(t) = 10(0.9548)^t$$
$$t = \frac{\ln 0.3}{\ln 0.9548}$$
$$t = 26.03 \text{ hours}$$

□

17. Convert to the form $Q = ae^{kt}$: $Q = 5.2(0.89)^t$. *Solution.* $Q = 5.2e^{-0.1165t}$ □

18. What is the equation of the asymptote of the graph

(a) $y = 3^{(-x)}$

Solution. $y = 0$ □

(b) $y = e^x$

Solution. $y = 0$ □

19. Find th hydrogen concentration $[H^+]$ for tomatoes which have a pH of 4.5 if $pH = -\log[H^+]$.
Solution. $[H^+] = 0.000032$ or $3.2x10^{-5}$. □

20. The figure below gives a graph for $f(x)$. Find a possible formula for $f(x)$.
Solution. $f(x) = \frac{-1}{2}(x + 1.5)(x - 4)$ □

21. Suppose that a quadratic function has its vertex at (3,0) and has y-intercept -4. Find a formula for the function.
Solution. $y = \frac{-4}{9}(x - 3)^2$ □

22. A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?
Solution. A formula for the area is $A(x) = x(4000 - 2x) = -2x^2 + 4000x$. Then a maximum would occur at $x = \frac{-4000}{-4} = 1000$. Thus the largest area enclosed is $A(1000) = 2,000,000$ square meters. □