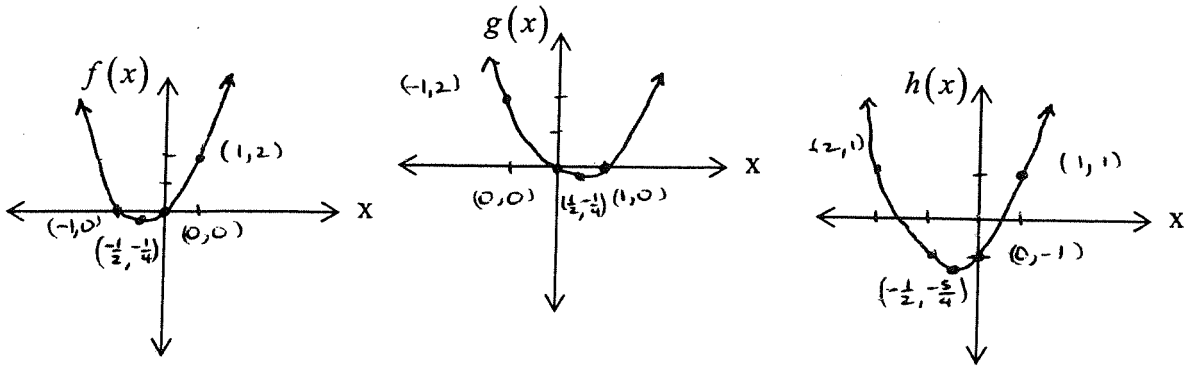
MA1032
Exam III Review
Fall 2003

1. Complete the following table using $f(x)=x^2+x$, $g(x)=f(x-1)$, and $h(x)=f(x)-1$.

x	-3	-2	-1	0	1	2	3
$f(x)$	6	2	0	0	2	6	12
$g(x)$		6	2	0	0	2	6
$h(x)$	5	1	-1	-1	1	5	11

shift right 1 unit so add 1 to each x $(-3,6)\to(-2,6)$
shift down 1 unit so subtract 1 from y $(-3,6)\to(-3,5)$

b) Graph the three functions, labeling 3 sets of coordinates.



c) Explain how the graphs of g and h are related to the graph of f.

g is f shifted to the right one unit
h is f shifted down one unit

2. Determine algebraically whether each of the following functions is even, odd, or neither.

a) $f(x)=x^2-5$

$f(-x)=(-x)^2-5$
$=x^2-5$
$f(-x)=f(x)$
even

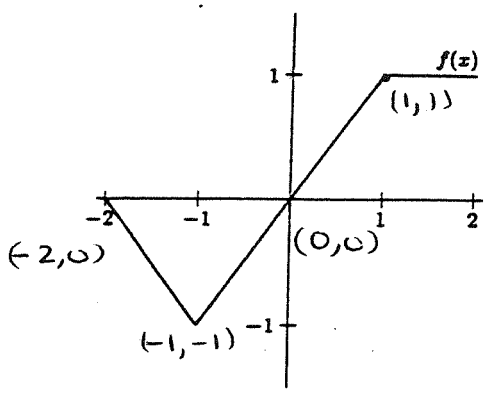
b) $g(x)=x^3-1$

$g(-x)=(-x)^3-1$
$=-x^3-1$
$=-(x^3+1)$
$g(-x)\neq g(x)\neq -g(x)$ neither

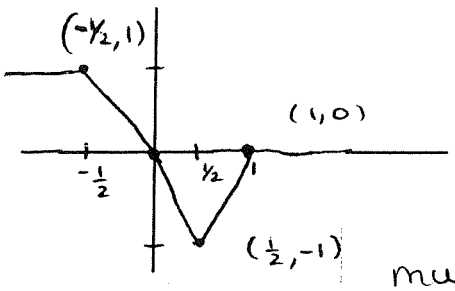
c) $h(x)=5x^3-x$

$h(-x)=5(-x)^3-(-x)$
$=-5x^3+x$
$=-5(x^3-x)$
$h(-x)=-h(x)$
odd

3. Using the figure below, graph and label 3 sets of coordinates for:



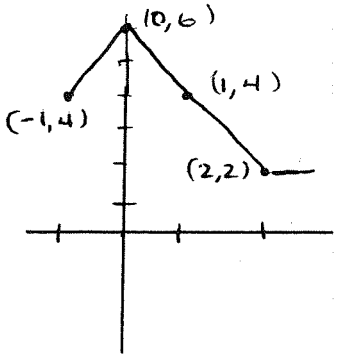
a) $y = f(-2x)$



multiply x by $-\frac{1}{2}$ (reflection, compression)

$(-2,0) \rightarrow (1,0)$
$(-1,-1) \rightarrow (\frac{1}{2},-1)$
$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (-\frac{1}{2},1)$

b) $y = -2f(x-1)+4$



add 1 to x, multiply y by -2 then add 4 (shift right 1, reflection & stretch vertically)

$(-2,0) \rightarrow (-1,4)$
$(-1,-1) \rightarrow (0,6)$
$(0,0) \rightarrow (1,4)$
$(1,1) \rightarrow (2,2)$

4. Let $g(x) = \frac{1}{x^2}$.

a) Give the formula for a function that transforms the graph of g as follows:

Shifts three units to the right, reflects about the x-axis, then shifts up two units.

$$y = -\frac{1}{(x-3)^2} + 2$$

b) Give the domain of the new function.

$(x-3)^2 \neq 0$
$x - 3 \neq 0$
$x \neq 3$

domain = $\{x \mid x \neq 3\}$

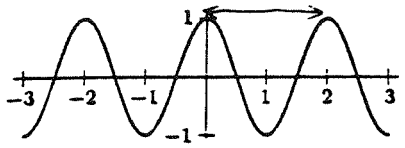
5 Estimate the period of the following periodic functions.

a)

t	π	2π	3π	4π	5π	6π	7π	8π	9π
$h(t)$	-1	0	1	0	-1	0	1	0	-1

$5\pi - \pi = 4\pi$

b)



2

6 The proposed London ferris wheel will be 500 feet in diameter and will make one revolution every 20 minutes. Let $y = h(t)$ be the height above ground in terms of t minutes of riding.

a) What is the period, the amplitude, and the midline for $h(t)$?

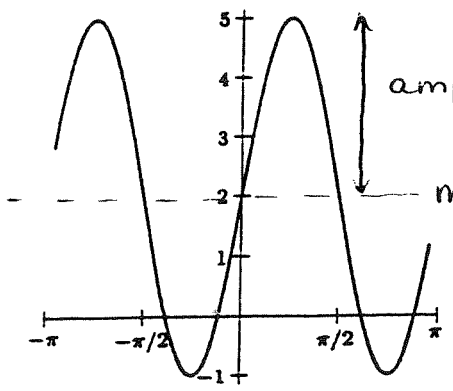
period = 20 min
$|A|$ = 250m
midline = 250m

b) Describe in a sentence or two what the following expressions would mean in terms of the London ferris wheel (for example, bigger wheel or faster wheel, etc.).

$h(t)+5$ – you board 5 m higher – loading platform is 5 ft off the ground

$h(t+5)$ – add 5 minutes to time; height of someone who boards 5 minutes before you.

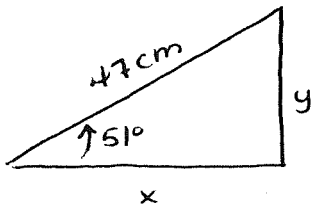
7 Find the midline and amplitude of the periodic function below.



amplitude = 5 - 2 = 3

midline: $y = 2$

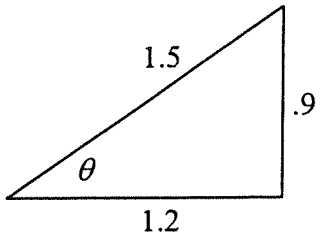
8 A right triangle has an acute angle A with a measurement of 51°. The hypotenuse of the triangle is 47 cm. Find the lengths of the other 2 sides.



$\cos 51^\circ = \frac{x}{47}$ $\quad 47\cos 51^\circ = x$ $\quad 29.6 = x$

$\sin 51^\circ = \frac{y}{47}$ $\quad 47\sin 51^\circ = y$ $\quad 36.5 = y$

9 A right triangle has sides as shown in the diagram. Give the values of the 6 trigonometric functions for the angle θ.



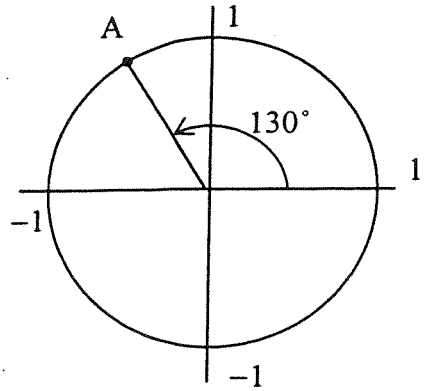
$\sin\theta = \frac{0.9}{1.5} = .6$ $\quad \csc\theta = \frac{1.5}{0.9} = 1.67$

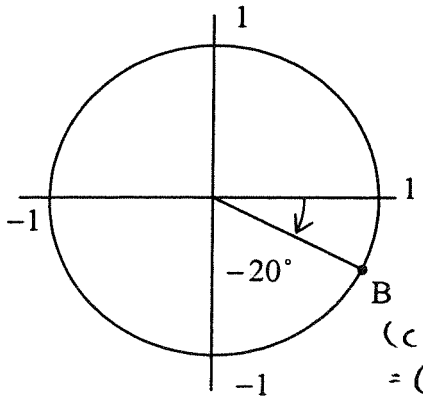
$\cos\theta = \frac{1.2}{1.5} = .8$ $\quad \sec\theta = \frac{1.5}{1.2} = 1.25$

$\tan\theta = \frac{0.9}{1.2} = .75$ $\quad \cot\theta = \frac{1.2}{0.9} = 1.3$

10 Find the coordinates of points A and B.

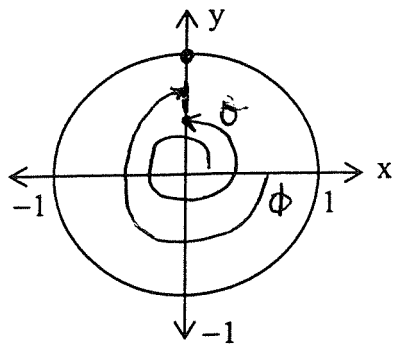
$(\cos 130^\circ, \sin 130^\circ) = (-0.643, 0.766)$





$(\cos -20^\circ, \sin -20^\circ)$
$= (0.9397, -0.342)$

11 Draw the point on the unit circle corresponding to the given angles and state the coordinates of each.



A: $\theta = 450°$ $\quad A = (\underline{0}, \underline{1})$

B: $\varnothing = -270°$ $\quad B = (\underline{0}, \underline{1})$

12 The coordinates of a point on a non-unit circle are $(-5.2623, -1.9153)$. The point is associated with an angle $\theta = 200°$. What is the radius of the circle?

$r\cos 200° = -5.2623$ $\quad$ or $\quad r\sin 200° = -1.9153$

$r = 5.6$ $\quad$ $r = 5.6$

13 Non-calculator: Fill in the blanks in the table given below.

degrees	0°	30°	135°	60°	270°	210°	720°	315°
radians	0	$\frac{\pi}{6}$	$\frac{3\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{2}$	$\frac{7\pi}{6}$	4π	$\frac{7\pi}{4}$

14 Calculator: a) Change to radians: $137° \left(\frac{\pi}{180}\right) = 2.3911$ radians

b) Change to degrees: $2.5 \left(\frac{180}{\pi}\right) = 143.24°$

15 Calculator: A circle with radius 3.12 contains an arc of length 8.6. Find the angle θ which spans the arc.

$s = r\theta$ $\quad \theta$ is in radians

$8.6 = 3.12\theta$

$2.756 = \theta$

radians

16 Non calculator: Sketch the graphs of the following functions. Give amplitude, period, midline equation, horizontal shift, and vertical asymptotes where possible. Clearly label x- and y- axes to show these features.

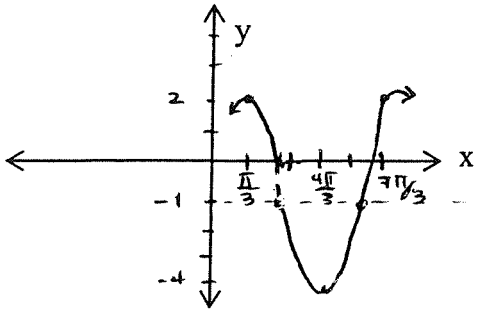
a) $y = 3\cos\left(x - \frac{\pi}{3}\right) - 1$

$|A| = 3$

period $= \frac{2\pi}{1} = 2\pi$

$h = \frac{\pi}{3}$

$k = -1$



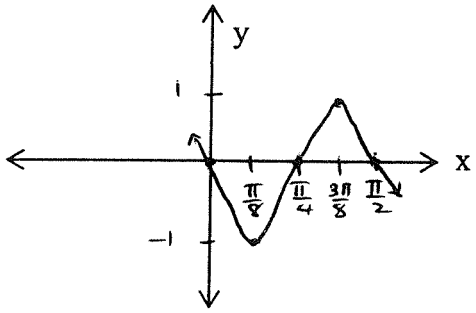
b) $y = -\sin(4x)$

$|A| = 1$

period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$h = 0$

$k = 0$

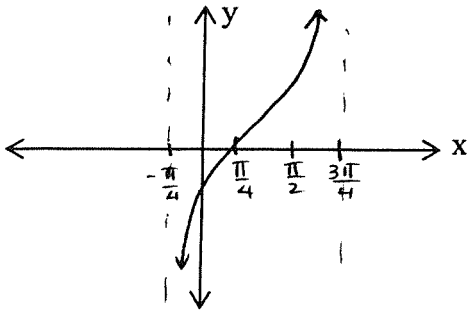


c) $y = \tan\left(x - \frac{\pi}{4}\right)$

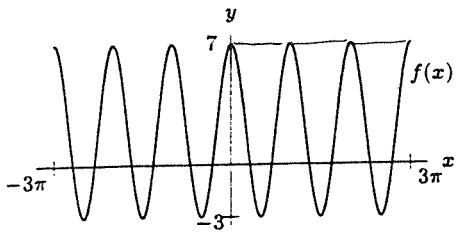
period $= \pi$

$h = \pi/4$

$k = 0$



17 Non calculator: Find a formula for the following graph.



midline = 2 = k starts at max so use cosine

|A| = 5

period = $\pi = \frac{2\pi}{B}$

B = 2

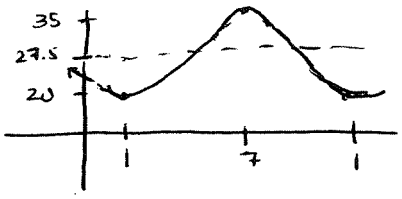
h = 0

$y = 5\cos(2x) + 2$

18 Calculator: Use a sinusoidal function to model the following situation.

The depth of the water at the end of an ocean pier oscillates. It has a low of 20 feet at 1 a.m. and a high of 35 feet at 7 a.m. At 1 p.m., there's another low of 20 feet; then the cycle repeats.

a) Find a formula for the depth of the water at time, t, measured in hours since midnight.



period = 12 = $\frac{2\pi}{B}$

B = $\frac{\pi}{6}$

midline = 27.5 = k

|A| = 7.5

h = 1

starts at min so use cosine with reflection

depth = $-7.5\cos\left(\frac{\pi}{6}(t-1)\right)+27.5$

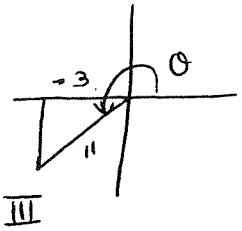
b) What is the height of the water at 3 o'clock?

$h(3) = -7.5\cos(\pi/6\,(2)) + 27.5$

$= 23.75$ ft

19 Non-calculator: θ is an angle in QIII. $\cos\theta = -\frac{3}{11}$. Find the values of the other 5 trigonometric functions of θ.

$\cos\theta = -\frac{3}{11} = \frac{x}{r}$



$(-3)^2 + y^2 = 11^2$

$y^2 = 112$

$y = -4\sqrt{7}$

since in QIII

$\sin\theta = \frac{y}{r} = \frac{-4\sqrt{7}}{11}$

$\tan\theta = \frac{y}{x} = \frac{4\sqrt{7}}{3}$

$\sec\theta = \frac{r}{x} = -\frac{11}{3}$

$\csc\theta = \frac{r}{y} = \frac{11}{-4\sqrt{7}} = \frac{-11\sqrt{7}}{28}$

$\cot\theta = \frac{x}{y} = \frac{3}{4\sqrt{7}} = \frac{3\sqrt{7}}{28}$

20 Non calculator: Evaluate

a) $\arcsin\left(\frac{1}{2}\right)$ where $-\pi/2 \le \theta \le \pi/2$

$\sin^{-1}\left(\frac{1}{2}\right) = \theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}$

b) $\cos^{-1}(-1)$ where $0 \le \theta \le \pi$

$\cos^{-1}(-1) = \theta$

$-1 = \cos\theta$

$\pi = \theta$

c) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ where $-\pi/2 \le \theta \le \pi/2$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$

$-\frac{\sqrt{3}}{2} = \sin\theta$

$-\frac{\pi}{3} = \theta$

d) $\sin\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$

$\sin\theta$ where $\theta = \sin^{-1}\frac{\sqrt{2}}{2}$; $-\pi/2 \le \theta \le$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$\sin \pi/4 = \frac{\sqrt{2}}{2}$

21 Non calculator: Find solutions for the equations such that $0 \le t \le 2\pi$.

a) $2\sin(2t) = 1$

$\sin 2t = \frac{1}{2}$

$2t = \pi/6, 5\pi/6$

$t = \pi/12 + \pi k$ $t = 5\pi/12 + \pi k$

$t = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$

b) $\cos^2 t = \frac{3}{4}$

$\cos t = \pm\frac{\sqrt{3}}{2}$

$t = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

22 State the reference angle for

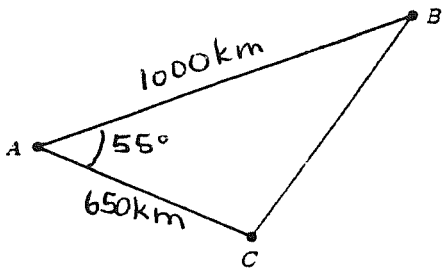
a) $\frac{4\pi}{3}$ $\frac{\pi}{3}$

b) $-\frac{5\pi}{6}$ $\frac{\pi}{6}$

c) $\frac{7\pi}{4}$ $\frac{\pi}{4}$

23 The boundaries of the Bermuda Triangle are not universally agreed upon, but one version places one corner of the triangle on the southern US coast (Point A in the figure), another corner in the Bermuda Islands (point B), and the third corner in the Greater Antilles (point C). Assume that the distance between A and B is 1000 km, and the angle at point A is 55°. Traveling at 25/ km/hr, a cruise ship takes 26 hours to cross from point A to point C. If the ship survives this portion of the trip, what is the distance it must travel from point C to point B?

SAS → Law of Cosines



$|AC| = (26\,hr)(25\,km/hr) = 650km$

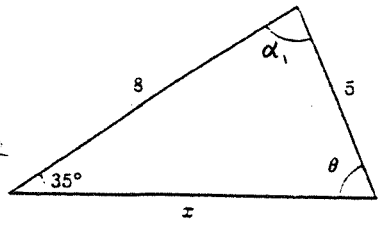
$|BC|^2 = 1000^2 + 650^2 - 2(1000)(650)(\cos 55°)$

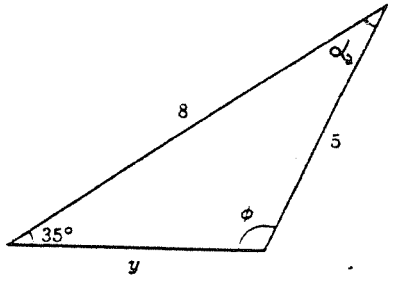
$= 676851$

$|BC| = \sqrt{676851} = 822.71km$

24 Let θ and ϕ be the two angles shown below and suppose $0 < \theta < 90°$ and $90° < \phi < 180°$.

SSA → Law of Sines





SSA → Law of Sines

a) Find θ and ϕ.

$\frac{\sin\theta}{8} = \frac{\sin 35°}{5}$ $\quad \theta = 66.6°$ since $0 < \theta < 90°$

$\sin\theta = \frac{8\sin 35°}{5}$

$\frac{\sin\phi}{8} = \frac{\sin 35°}{5}$

$\sin\phi = \frac{8\sin 35°}{5} = 66.6°$

but $\phi = 180° - 66.6°$

$\phi = 113.4°$

since $90° < \phi < 180°$

b) Find x and y.

$\alpha_1 = 180° - (35° + 66.6°) = 78.4°$

$\frac{\sin\alpha_1}{x} = \frac{\sin 35°}{5}$

$x = \frac{5\sin\alpha_1}{\sin 35°} = \frac{5\sin(78.4°)}{\sin 35°} = 8.539$

$\alpha_2 = 180° - (35° + 113.4°) = 31.6°$

$\frac{\sin\alpha_2}{y} = \frac{\sin 35°}{5}$

$y = \frac{5\sin\alpha_2}{\sin 35°} = \frac{5\sin 31.6°}{\sin 35°}$

$= 4.568$

MA1032
Formulas

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$