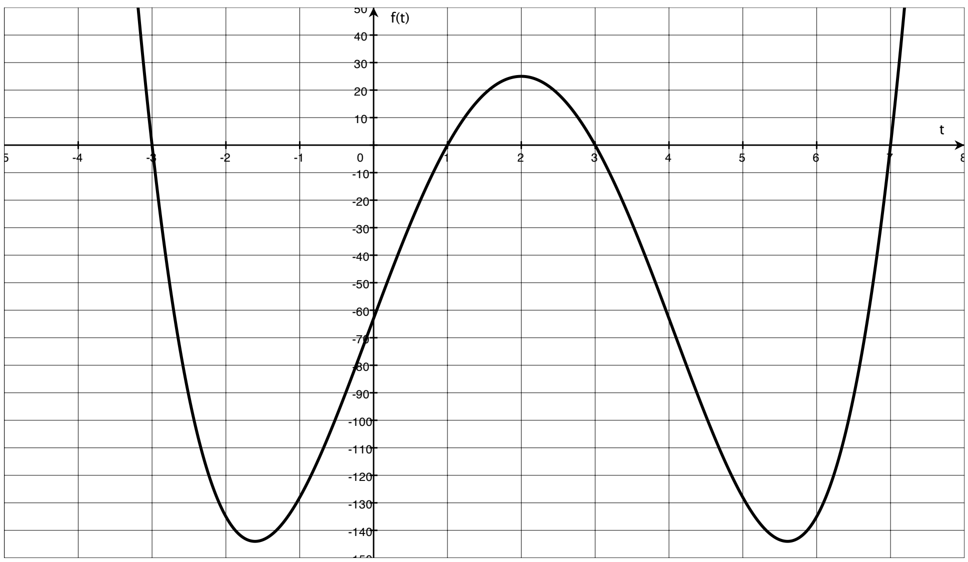
MA1032 – Exam 1 – Solutions

Name____

September 28, 2006

1. On approximately what intervals is the function below increasing? Decreasing?



Solution. The function is increasing for approximately $-1.5 \leq t \leq 2$ and $5.5 \leq t \leq \infty$. The function is decreasing for approximately $-\infty \leq t \leq -1.5$ and $2 \leq t \leq 5.5$. □

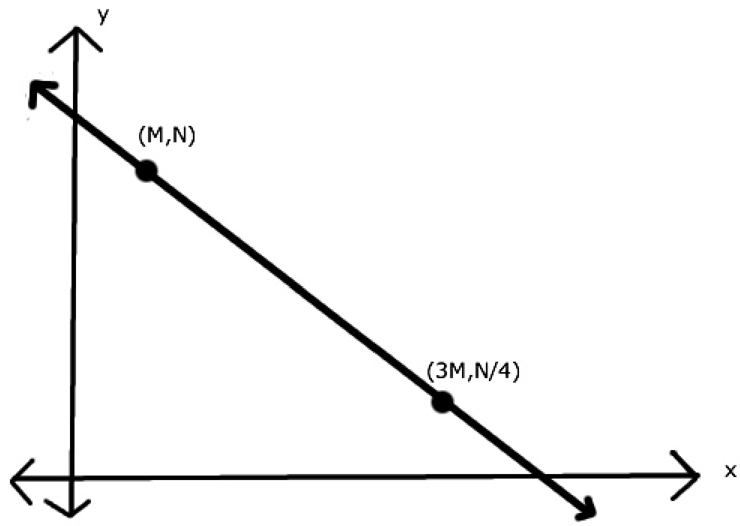
2. Suppose that the following table shows the cost of a taxi ride, in dollars, as a function of miles traveled.

m	0	1	2	3	4	5
$C(m)$	0	2.5	4.00	5.50	7.00	8.50

(a) What does $C(3.5)$ mean in practical terms? Estimate $C(3.5)$.
Solution. $C(3.5)$ is the cost of 3.5 mile taxi ride. $C(3.5) \approx 6.25$. □

(b) If $C(m) = 3.5$, what does m mean in practical terms? Estimate m.
Solution. In the context of $C(m) = 3.5$, m is the number of miles you can ride for \$3.5. $m \approx 1.75$. □

3. Find an equation for the line shown in the following figure. Your equation will involve the constants given on the graph. Be sure to simplify your answer.



Solution. The slope is

$$m = \frac{N - \frac{N}{4}}{M - 3M} = \frac{\frac{3N}{4}}{-2M} = \frac{-3N}{8M}.$$

Using point slope form, we get $y - N = \frac{-3N}{8M}(x - M)$.
Simplifying, we get $y = \frac{-3N}{M}x + \frac{11N}{8}$. □

4. Suppose $f(x) = x^2$.

   (a) Find the average rate of change of the function f between $x = 1$ and $x = 4$.
   Solution.

$$\frac{\Delta f}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = \frac{15}{3}$$

□

   (b) Find the value of c making the average rate of change between $x = 1$ and $x = c$ equal to 10.
   Solution.

$$\frac{\Delta f}{\Delta x} = \frac{f(c) - f(1)}{c - 1} = \frac{c^2 - 1}{c - 1} = 10$$

$$c^2 - 1 = 10(c - 1)$$

$$c^2 - 10c + 9 = 0$$

Using the quadratic formula, you get

$$c = \frac{10 \pm \sqrt{100 - 36}}{2} = \frac{10 \pm 8}{2} = 9, 1.$$

Since $c = 1$ doesn't make sense in the context of the problem, our answer is $c = 9$. $\square$

5. (a) Find the domain of $r(x) = \frac{1}{(x+2)^2} + \sqrt{1 - x}$.
   Solution. The domain is all real values of x such that $x < 1$ and $x \neq -2$. $\square$

   (b) Find the range of $s(x) = \frac{6-x}{3+x}$.
   Solution. The range is all real values of $s(x)$ such that $s(x) \neq -1$. $\square$

6. A T-shirt printing company charges a set-up fee of \$10 for each order, plus the cost per shirt show in the following table.

# of shirts	cost per shirt
0-10	\$10
11-20	\$9
21-30	\$8
over 30	\$7

   (a) Express C, the total cost in dollars, as a piecewise function of n, the number of shirts ordered.
   Solution.

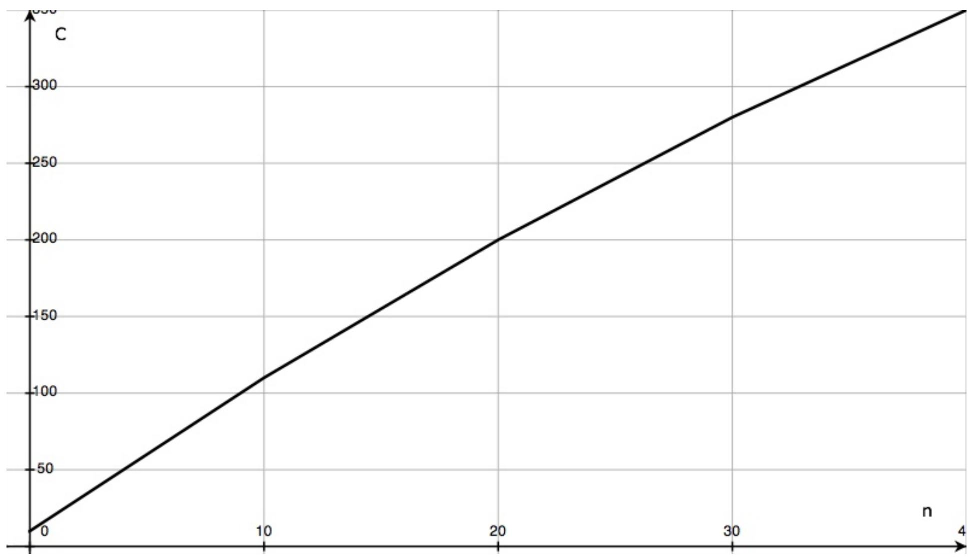
$$C(n) = \begin{cases} 10 + 10n & \text{if } 0 \leq n \leq 10, \\ 110 + 9(n - 10) & \text{if } 10 < n \leq 20, \\ 110 + 90 + 8(n - 20) & \text{if } 20 < n \leq 30, \\ 110 + 90 + 80 + 7(n - 30) & \text{if } n > 30. \end{cases}$$

Though not necessary, it will make part (b) easier. Simplification yields,

$$C(n) = \begin{cases} 10 + 10n & \text{if } 0 \leq n \leq 10, \\ 20 + 9n & \text{if } 10 < n \leq 20, \\ 40 + 8n & \text{if } 20 < n \leq 30, \\ 70 + 7n & \text{if } n > 30. \end{cases}$$

$\square$

(b) Sketch a graph of C for $0 \leq n \leq 40$.
Solution.



$\square$

7. The circumference, in cm, of a circle whose radius is r cm is given by $C = 2\pi r$.

   (a) Write this formula using function notation, where f is the name of the function.
   Solution. $C = f(r) = 2\pi r$. $\square$

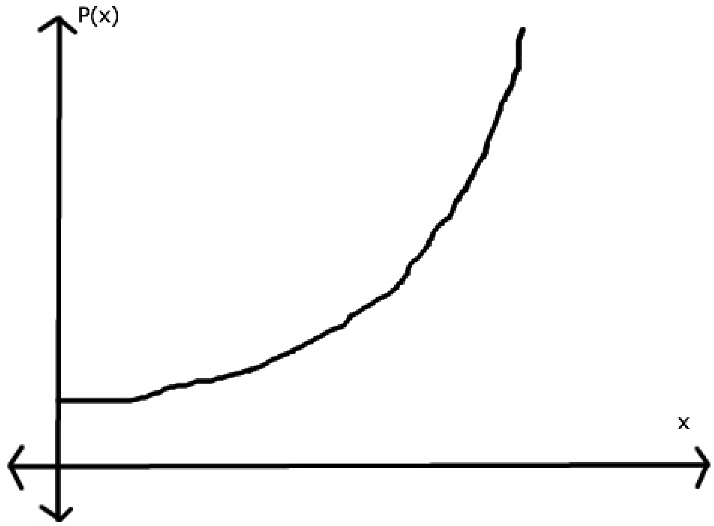
   (b) Evaluate and interpret $f(r + 2)$.
   Solution. $f(r + 2) = 2\pi(r + 2) = 2\pi r + 4\pi$. This means that if we increase the radius by 2, we end up increasing the circumference by 4π. $\square$

   (c) Evaluate and interpret $f(r) + 2$.
   Solution. $f(r)+2 = 2\pi r+2$. So $f(r)+2$ represents increasing the circumference by 2. $\square$

   (d) Evaluate and interpret $f^{-1}(8\pi)$.
   Solution. Since solving $8\pi = 2\pi r$ yields $r = 4$, we know that $f^{-1}(8\pi) = 4$. So the notation $f^{-1}(8\pi)$ represents the radius of a circle with circumference 8π. $\square$

8. The probability of being in an accident increases as a driver's blood-alcohol content (BAC) rises. It has been observed that the probability rises faster and faster as the BAC increases. Let $P(x)$ be the probability that a driver will be involved in an accident as a function of x, the driver's BAC.

   (a) Sketch a graph of $P(x)$ and label the axes.
   Solution.



$\square$

   (b) Clearly indicate on your above graph whether or where your graph is increasing, decreasing, concave up, or concave down.
   Solution. The graph is *everywhere* increasing and concave up. $\square$