MA1032 – Workshop – Chapter 1

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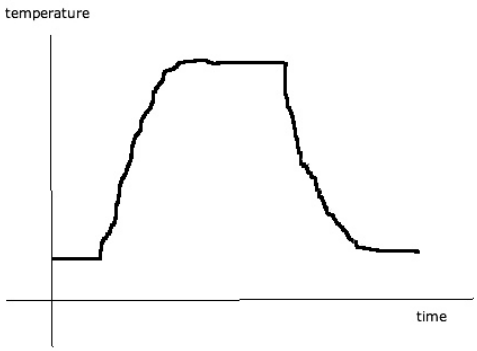
1. Is y a function of x? Is x a function of y. Explain. (See graph on your worksheet.)

   Solution.y is not a function of x because for one x value there are two y values. x is not a function of y because the graph shown on the worksheet does not satisfy the horizontal line test, that is, for some y values there are two x values. □

2. A light is turned off for several hours. It is then turned on. After a few hours it is turned off again. Sketch the light bulb's temperature asa function of time. Label the axes.

   Solution.



□

3. Explain why the table below represents a linear function and find a formula for the function.

t	3	6	9	12	15
$a(t)$	2	4	6	8	10

Solution. Linear functions have a constant rate of change. Therefore, since $\frac{\Delta a(t)}{\Delta t} = \frac{2}{3}$ for all consecutive points, the table represents a linear function. Then $a(t) = b + \frac{2}{3}t$. Plugging in a point, say $(3, 2)$, we get $b = 0$. Thus $a(t) = \frac{2}{3}t$. □

4. A cylindrical water tank has a base of diameter 6 feet.

   (a) Write a formula for the volume of water as a function of its depth in the tank.
   Solution. $V(d) = \pi r^2 d$. Since $r = 3$, $V(d) = 9\pi d$. □

   (b) How much water can the tank hold if the tank is 8 feet tall?
   Solution. $V(8) = 9\pi d$. Thus $V(8) = 72\pi$ ft^3. □

5. A small café sells coffee for \$0.95 per cup. On average, it costs the café \$0.25 to make a cup of coffee (for grounds, hot water, filters). The café also has a fixed daily cost of \$200 (for rent, wages, utilities).

   (a) Let R, C, and P be the café's daily revenue, costs, and profit, respectively, for selling x cups of coffee in a day. Find formulas for R, C, and P as a function of x. [Hint: The revenue, R is the total amount of money that the café brings in. The cost, C, includes the fixed daily cost as well as the cost for all x cups of coffee sold. P is the café's profit after costs have been accounted for.]
   Solution.

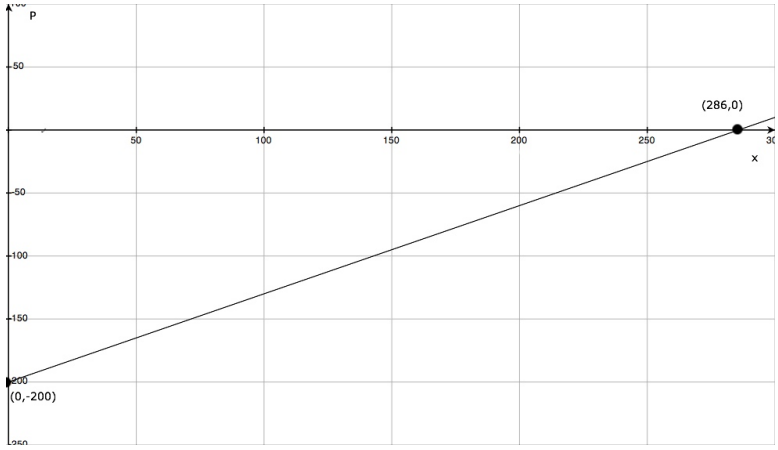
$$R(x) = 0.95$$

$$C(x) = 200 + 0.25x$$

$$P(x) = R(x) - C(x) = 0.95x - (200 + 0.25x) = 0.70x - 200$$

   □

   (b) Plot P against x. Clearly label both the x- and y- intercepts on the graph.
   Solution.



□

(c) Interpret the slope and both intercepts of your graph in practical terms.

*Solution.*The slope is 0.70. This represents a 70 cent increase in profit for each cup of coffee sold. The x-intercept is about 286. This represents the breakeven point, i.e., 286 cups need to be sold in order to make a profit. The y-intercept is -200. If you don't sell any cups of coffee, you lose \$200. □

6. The table below gives the cost, $C(n)$, of producing a certain good as a linear function of n, the number of units produced.

n (units)	100	125	150	175
$C(n)$ (dollars)	11,000	11,125	11,250	11,375

Evaluate the following expressions and **give economic interpretations for each.**

(a) $C(175)$

*Solution.*The cost of producing 175 units is \$11,375. □

(b) $C(175) - C(150)$.

Solution.$C(175) - C(150) = 11,375 - 11,250 = 125$. The cost of producing the next 25 units (after 150 units) is \$125. It costs \$125 more to produce 175 units than to produce 150 units. □

(c) $\frac{C(175)-C(150)}{175-150}$

Solution.$\frac{C(175)-C(150)}{175-150} = \frac{125}{25} = 5$. The cost of production increases by \$5 for each additional unit produced. □