Inverse Trigonometry Worksheet

Name ______________

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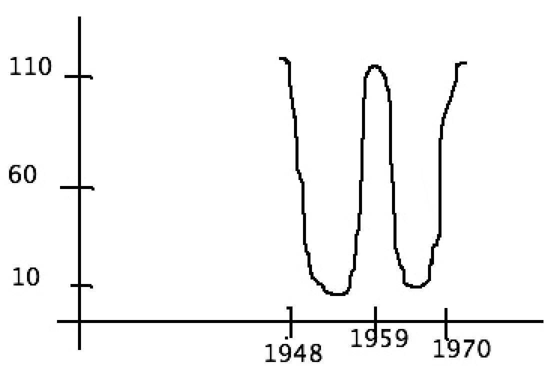
1. For several hundred years astronomers have kept track of the number of solar flares, or "sunspots," which occur on the surface of the sun. The number of sunspots counted in a given year varies sinusoidally from a minimum of about 10 per year to a maximum of about 110 per year. Between the maximums that occurred in the years 1750 and 1948, there were 18 complete cycles.

   (a) What is the period of the sunspot cycle?
   Solution. $\frac{1948-1750}{18} = 11$ □

   (b) Assume a sinusoidal function $s(x)$ computes the number of sunspots in a given year x. Sketch the graph for two sunspot cycles, starting from 1948.
   Solution. Assuming that $x = 0$ corresponds to "year zero," you get the following rough sketch.



□

   (c) What is the amplitude of the sunspot cycle?
   Solution. $A = \frac{110-10}{2} = 50$ □

(d) What is the mean (D) of the sunspot cycle?

Solution. $D = \frac{110+10}{2} = 60$ □

(e) Given that $C = \frac{-7}{4}$ and that a sinusoidal function is of this form $s(x) = A\sin(\frac{2\pi}{B}(x - C)) + D$, give the sinusoidal equation for the number of sunspots in a given year x.

Solution. Notice here that the form of $s(x)$ is given a bit different from what I did in class on Thursday. B is used as the period and not as we used it in class. In class, we would have gotten $B = \frac{2\pi}{11}$ which will come out the same in the equation $y = 50\sin(\frac{2\pi}{11}(x + C)) + 60$. C is also different, I used $+C$ in the equation, this worksheet used $-C$. All then that changes is the sign. Therefore the value for C that corresponds to the form we used in class would be $\frac{7}{4}$. But the value we got in class was $-1750 + \frac{11}{4} = \frac{-6989}{4}$ not $\frac{7}{4}$. What happened? Remember that the value of C is not unique. There are an infinite number of shifts which produce the same graph. What is unique about the values for C is that you can add or subtract multiples of the period to one value and come up with the other value. Therefore the following should hold.

$$\frac{7}{4} - 11m = -\frac{6989}{4}, \quad \text{where } m \text{ is a whole number.}$$

If we solve for m and get a whole number, then we know the value we got in class is also correct. Solving the above equation yields $m = 159$, a whole number. Therefore, using $C = -\frac{6989}{4}$ will produce the same graph as $C = \frac{7}{4}$. You can check this by graphing on your calculator,

$$s(x) = 50\sin(\frac{2\pi}{11}(x - \frac{6989}{4})) + 60$$

and

$$s(x) = 50\sin(\frac{2\pi}{11}(x + \frac{7}{4})) + 60.$$

The graphs will be identical.

□

(f) How many sunspots would you expect in the year 2000?

Solution.

$$s(2000) = 50\sin(\frac{2\pi}{11}(2000 - \frac{6989}{4})) + 60 \approx 52.8843$$

Therefore, you would expect about 53. □

(g) Given that the principle solution to $s(x) = 35$ is $x = \frac{-8}{3}$, find the first year after the year 2000 in which the number of sunspots will be about 35.

Solution. Remember our shift was different by 159 periods! Therefore, a reasonable prediction is that the solution $x = -\frac{8}{3}$ for $s(x) = 35$ is also different from our solution by 159 periods. Let's check. Solving the equation we got in class goes as follows.

$$50\sin(\frac{2\pi}{11}(x-\frac{6989}{4}))+60=35$$
$$\sin(\frac{2\pi}{11}(x-\frac{6989}{4}))=-\frac{1}{2}$$
$$\frac{2\pi}{11}(x-\frac{6989}{4})=\arcsin(-\frac{1}{2})$$
$$x-\frac{6989}{4}=\frac{11}{2\pi}\arcsin(-\frac{1}{2})$$
$$x=\frac{11}{2\pi}\arcsin(-\frac{1}{2})+\frac{6989}{4}$$
$$x=\frac{5239}{3}$$

How many periods is this different from the x value on the worksheet?

$$-\frac{8}{3}+11m=\frac{5239}{3}$$
$$m=159$$

Just as we suspected! We can use both equations with their solutions, respectively, to solve the same problem. Okay, so the first year after 2000 for 35 sunspots? We need to first find all solutions (we only found one so far) to $s(x)=35$. The closest max to $\frac{5239}{3}\approx 1746.33$ is 1750. By symmetry, we need to add the distance $1750-\frac{5239}{3}$ to 1750 to get to the solution on the other side of the max. That is, $1750+(1750-\frac{5239}{3})=\frac{5261}{3}\approx 1753.67$. Therefore, all solutions are

$$x=\frac{5239}{3}\pm 11k \text{ and } \frac{5261}{3}\pm 11k, \text{ where } k=0,1,2,\ldots$$

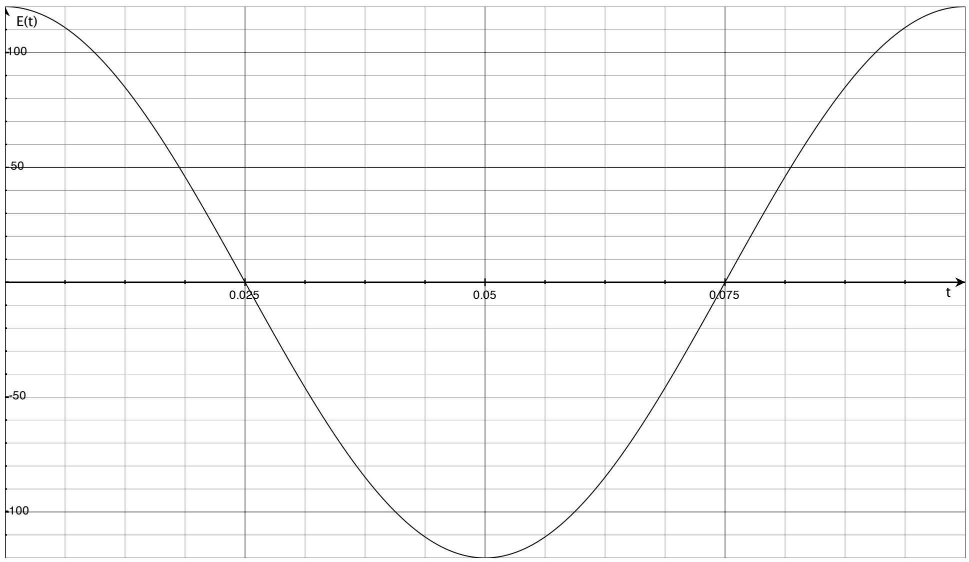
To find the first value after 2000 we just need to add 11 to both of the answers enough times to get to the value that is closest to 2000, yet still above. After adding 11 a bunch of times to $\frac{5239}{3}$, the first value that surpasses 2000 is approximately 2010.33. After adding 11 a bunch of times to $\frac{5261}{3}$, the first value that surpasses 2000 is approximately 2006.67. Therefore, 2006 is the first year after 2000 in which the number of sunspots is about 35. □

2. The voltage E of a circuit after t seconds ($t>0$) is given by the equation $E(t)=120\cos(20\pi t)$; angles are in radian measure.

(a) Find the period of $E(t)$.

Solution. $\frac{2\pi}{20\pi} = \frac{1}{10}$ □

(b) Sketch the graph of $E(t)$.



(c) Given that the principle solution to $E(t) = 10$ is $t = .02367$, find the times during the first second when the voltage is 10.

Solution. From the graph, we see that the min occurs at $\frac{1}{20}$. Therefore, the principal solution is on the left side of the min. The distance between the min and the principal solution is $\frac{1}{20} - .02367 \approx .02633$. Adding that same distance past the min in order to get the symmetric solution yields $\frac{1}{20} + .02633 \approx .07633$. These are the times within the first tenth of a second. There are 20 solutions within the first second. To get these just add periods to the principal and symmetric solutions.

$$t = 0.02367, 0.12367, 0.22367, 0.32367, 0.42367, 0.52367, 0.62367, 0.72367, 0.82367, 0.92367,$$

$$0.07633, 0.17633, 0.27633, 0.37633, 0.47633, 0.57633, 0.67633, 0.77633, 0.87633, 0.97633$$

□