Product & Quotient Rules Worksheet

Name ____________________

February 21, 2007

1. Find the derivatives of the given functions.

   (a) $f(x) = (3x^2 + 6)(2x - \frac{1}{4})$
   Solution. $f'(x) = 6x(2x - \frac{1}{4}) + 2(3x^2 + 6)$ □

   (b) $f(x) = (2 - x - 3x^3)(7 + x^5)$
   Solution. $f'(x) = (-1 - 9x^2)(7 + x^5) + (2 - x - 3x^3)(5x^4)$ □

   (c) $y = \frac{1}{5x-3}$
   Solution. $y' = \frac{-5}{(5x-3)^2}$ □

   (d) $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$
   Solution. $f(x)$ may be rewritten as $(x^{-1} + x^{-2})(3x^3 + 27)$.
   Then $f'(x) = (-x^{-2} - 2x^{-3})(3x^3 + 27) + (x^{-1} + x^{-2})(9x^2)$. □

   (e) $y = \frac{3}{\sqrt{x}+2}$
   Solution. $y' = \frac{-3(\frac{1}{2}x^{-1/2})}{(\sqrt{x}+2)^2}$ □

   (f) $x = \frac{3t}{2t+1}$
   Solution. $x' = \frac{(2t+1)3-3t(2)}{(2t+1)^2}$ □

   (g) $g(x) = \left(\frac{3x+2}{x}\right)(x^{-5} + 1)$
   Solution. $g(x)$ may be rewritten as $g(x) = (3 + 2x^{-1})(x^{-5} + 1)$.
   Then $g'(x) = (-2x^{-2})(x^{-5} + 1) + (3 + 2x^{-1})(-5x^{-6})$. □

   (h) $y = (t^3 - 7t^2 + 1)e^t$
   Solution. $y' = (3t^2 - 14t)e^t + (t^3 - 7t^2 + 1)e^t$. □

   (i) $y = \frac{t+1}{2^t}$
   Solution. $y' = \frac{2^t-(t+1)(\ln 2)2^t}{2^{2t}}$ □

   (j) $g(r) = r \cdot 2^r$
   Solution. $g'(r) = 2^r + r(\ln 2)2^r$ □

2. Suppose f and g are differentiable functions with the values shown in the following table. For each of the following functions h, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	-2

(a) $h(x) = f(x) + g(x)$

Solution. $h'(2) = f'(2) + g'(2) = 5 - 2 = 3$ □

(b) $h(x) = f(x)g(x)$

Solution. $h'(2) = f'(2)g(2) + f(2)g'(2) = 5 \cdot 4 + 3 \cdot (-2) = 14$ □

(c) $h(x) = \frac{f(x)}{g(x)}$

Solution. $h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{g(2)^2} = \frac{4 \cdot 5 - 3 \cdot (-2)}{4^2} = \frac{13}{8}$ □