

Fall 2006
MA1160-MA1161
FINAL EXAM – Part I (Calculators allowed)
December 19, 2006
Do all problems – show all work

Calculators are only allowed on Part I of this exam. Put away your calculators when you turn in this part of the exam.

Work on Part I first (it is the calculator part of this exam). It will be collected during the first third of the exam period and you will put away your calculators before proceeding with part II.

You may work on Part II of this exam at any time, but you must have first handed in part I and put away your calculators.

This part of the exam contains 3 pages and Problems 1 through 4. Make sure you have a complete test booklet.

Name _____ ID _____
 (please print)

Circle your section number and instructor's name:

<u>Course</u>	<u>Section</u>	<u>Instructor</u>	<u>Days and Time</u>
MA1160	R01	J. Bierbrauer	MWF 09:05
	R02	V. Tonchev	MWF 10:05
	R03	A. Niu	MWF 12:05
	R04	D. Olson	MWF 02:05
	R05	A. Roy	MWF 08:05
	R06	D. Olson	MWF 04:05
	R07	A. Roy	MWF 04:05
MA1161	R01	X. Cui	MTWR 08:05
	R02	B. Bertram	MTWR 09:05
	R03	T. Grassl	MTWR 11:05
	R04	L. Xiao	MTWR 12:05
	R05	C. Givens	MTWR 03:05
	R06	H. VanSpronsen	MTWR 02:05
	R07	K. Heuvers	MTWR 02:05

<u>Page</u>	<u>Score</u>
1	/ 15
2	/ 11
3	/ 12
<i>Subtotal of part I</i>	/ 38
<i>Subtotal of part II</i>	/ 162
TOTAL	/ 200

1. We are given the following data for the functions f and g and their derivatives.

$$\begin{array}{cccc} f(-1) = 7 & f'(-1) = -8 & g(-1) = -6 & g'(-1) = -2 \\ f(1) = 3 & f'(1) = -5 & g(1) = -3 & g'(1) = 4 \\ f(0) = 4 & f'(0) = 3 & g(0) = -1 & g'(0) = 2 \end{array}$$

a) If $H(x) = 3f(x) - 2g(x)$, find $H'(0)$.

$$H'(0) = \underline{\hspace{2cm}} \quad (3 \text{ pts})$$

b) If $K(x) = f(x)g(x)$, find $K'(0)$.

$$K'(0) = \underline{\hspace{2cm}} \quad (4 \text{ pts})$$

c) If $L(x) = f(g(x))$, find $L'(0)$.

$$L'(0) = \underline{\hspace{2cm}} \quad (4 \text{ pts})$$

d) If $M(x) = \frac{g(x)}{f(x)}$, find $M'(0)$.

$$M'(0) = \underline{\hspace{2cm}} \quad (4 \text{ pts})$$

page total: (15 pts)

2. Values of a function $y = f(x)$ are given in the following table for $0 \leq x \leq 10$.

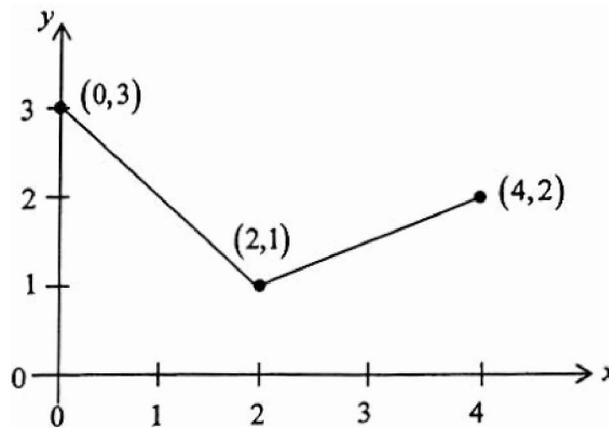
x	0	2	4	6	8	10
$f(x)$	2	12	30	62	70	72

Use a left-hand sum with five rectangles to estimate the value of $\int_0^{10} f(x) dx$.

(Show the formula you used to compute your answer.)

Left-hand sum = _____ (4 pts)

3. The graph of $y = f(x)$ for $0 \leq x \leq 4$ is shown in the figure below.



a) Use the graph to find $\int_0^4 f(x) dx$.

Answer: _____ (4 pts)

b) Find the average value of f on $[0, 4]$.

Answer: _____ (3 pts)

page total: _____ (11 pts)

4. You are given that $\int_a^b f(x) dx = 5$, $\int_a^b g(x) dx = 3$, $\int_a^b f^2(x) dx = 16$, $\int_a^b g^2(x) dx = 36$.
Find the following.

a) $\int_a^b (4f(x) - 3g(x)) dx$

Answer: _____ (4 pts)

b) $\int_a^b [4f^2(x) - 3g^2(x)] dx$

Answer: _____ (4 pts)

c) $4 \left[\int_a^b f(x) dx \right]^2 - 3 \left[\int_a^b g(x) dx \right]^2$

Answer: _____ (4 pts)

page total: _____ (12 pts)

Fall 2006
MA1160-MA1161
FINAL EXAM – Part II (NO calculators allowed)
December 19, 2006

Do all problems – show all work – partial credit will be based on work shown.

Work on part I first (calculator part of exam). It will be collected during the first third of the exam period and you will put away your calculator before proceeding with part II).

You may work on part II of this exam at any time, but you must have first handed in part I and put away your calculator.

This part of the exam contains 7 pages (pages 4 through 10) and Problems 5 through 15. Make sure you have a complete test booklet.

Name _____
 (please print)

ID _____

Circle your section number and instructor's name:

<u>Course</u>	<u>Section</u>	<u>Instructor</u>	<u>Days and Time</u>
MA1160	R01	J. Bierbrauer	MWF 09:05
	R02	V. Tonchev	MWF 10:05
	R03	A. Niu	MWF 12:05
	R04	D. Olson	MWF 02:05
	R05	A. Roy	MWF 08:05
	R06	D. Olson	MWF 04:05
	R07	A. Roy	MWF 04:05
MA1161	R01	X. Cui	MTWR 08:05
	R02	B. Bertram	MTWR 09:05
	R03	T. Grassl	MTWR 11:05
	R04	L. Xiao	MTWR 12:05
	R05	C. Givens	MTWR 03:05
	R06	H. VanSpronsen	MTWR 02:05
	R07	K. Heuvers	MTWR 02:05

<u>Page</u>	<u>Score</u>
4	/ 30
5	/ 18
6	/ 29
7	/ 22
8	/ 20
9	/ 16
10	/ 27
<i>Subtotal of part II</i>	/ 162

5. Find the derivatives of the following functions.

a) $f(x) = 4x^3 + e^x$

$f'(x) =$ _____ (6 pts)

b) $g(x) = 2 \cos(8x+3)$

$g'(x) =$ _____ (6 pts)

c) $h(x) = \ln(1+x^3)$

$h'(x) =$ _____ (6 pts)

d) $P(t) = 4e^{t^2}$

$P'(t) =$ _____ (6 pts)

e) $G(x) = \arcsin(e^x)$

$G'(x) =$ _____ (6 pts)

page total: _____ (30 pts)

6. Find the derivatives of the following functions.

a) $g(x) = x^2 \sin(x^3)$

$g'(x) =$ _____ (6 pts)

b) $h(x) = \int_0^x \cos(2t^4) dt$

$h'(x) =$ _____ (6 pts)

7. Find an equation of the tangent line to the graph of $f(x) = 2 + 3 \sin(x + \pi)$ at the point corresponding to $x = 0$.

An equation of the tangent line: _____ (6 pts)

page total: _____ (18 pts)

8. Find the values of the following integrals.

a) $\int (8t^3 + 3t^2) dt$

Answer: _____ (6 pts)

b) $\int_1^2 (8t^3 + 3t^2) dt$

Answer: _____ (6 pts)

9. For each of the following find an antiderivative.

a) $g(x) = \frac{12}{x} + 2 \cos x$

Answer: _____ (6 pts)

b) $h(x) = 12 \sin(3x) + \frac{1}{\cos^2 x}$

Answer: _____ (6 pts)

c) $G(x) = 7^x$

Answer: _____ (5 pts)

page total: _____ (29 pts)

10. A function $f(x) = -(x+3)(x-2)^2$ is defined for $-\infty < x < \infty$ and has $f'(x) = -3(3x+4)(x-2)$ and $f''(x) = -6(3x-1)$.

a) Identify the intervals on the x -axis where $f(x)$ is positive.

Answer: _____ (4 pts)

b) Identify the intervals on the x -axis where $f'(x)$ is positive.

Answer: _____ (4 pts)

c) Identify the intervals on the x -axis where $f''(x)$ is positive.

Answer: _____ (4 pts)

11. The position of a moving particle on a straight line is given by $s = s(t) = 6t^3 + 8t^2 + 3t + 5$ where distance is measured in meters and time t is measured in seconds.

a) Find the instantaneous velocity v when $t = 1$ (include units).

$v(1) =$ _____ (5 pts)

b) Find the average velocity between $t = 0$ and $t = 2$ (include units).

average velocity = _____ (5 pts)

page total: _____ (22 pts)

12. The point $A = (1,1)$ lies on the curve

$$x^4 + xy^2 + y^3 = 3.$$

a) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y .

$$\frac{dy}{dx} = \underline{\hspace{10cm}} \quad (6 \text{ pts})$$

b) Find an equation for the tangent line to this curve at the point $A = (1,1)$.

An equation for the tangent line: $\underline{\hspace{10cm}}$ (6 pts)

13. The surface area of a sphere of radius r is $A = 4\pi r^2$. If the radius of sphere is increasing at a rate of 3 cm/min, find the rate at which the surface area A is increasing when $r = 5$ cm. (Include units).

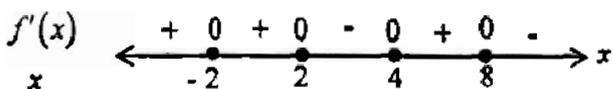
$$\frac{dA}{dt} = \underline{\hspace{10cm}} \quad (8 \text{ pts})$$

(include units)

page total: $\underline{\hspace{10cm}}$ (20 pts)

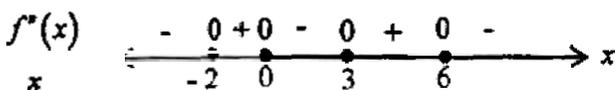
14. You know that a function $y = f(x)$ with a continuous second derivative and defined for all

x has a 1st derivative $f'(x)$ satisfying



$f'(x)$ $\leftarrow \begin{array}{ccccccccccc} & + & 0 & + & 0 & - & 0 & + & 0 & - & \\ & & -2 & & 2 & & 4 & & 8 & & \\ & & & & & & & & & & \end{array} \rightarrow x$

and a 2nd derivative $f''(x)$ satisfying



$f''(x)$ $\leftarrow \begin{array}{ccccccccccc} & - & 0 & + & 0 & - & 0 & + & 0 & - & \\ & & -2 & & 0 & & 3 & & 6 & & \\ & & & & & & & & & & \end{array} \rightarrow x$

a) Identify the intervals on the x -axis where $f(x)$ is decreasing.

Answer: _____ (4 pts)

b) Identify the x values of the points where $f(x)$ has local maxs.

Answer: _____ (4 pts)

c) Identify the intervals on the x -axis where $y = f(x)$ is concave up.

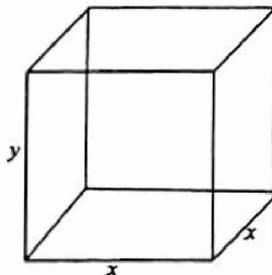
Answer: _____ (4 pts)

d) Give the x values where $f(x)$ has points of inflection.

Answer: _____ (4 pts)

page total: _____ (16 pts)

15. You are required to construct a box with a square base with side x *cms* and altitude y *cm*. The box has no top (see diagram). The box has a fixed volume $V = 4000 \text{ cm}^3$.

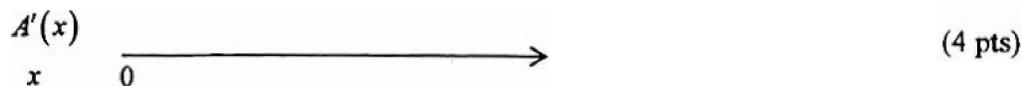


- a) Find a formula for the surface area $A = A(x)$ as a function of x .

$$A(x) = \underline{\hspace{10em}} \quad (8 \text{ pts})$$

- b) Find $A'(x)$ and indicate on the number line where it is positive (+), negative (-), and zero (0) for $0 < x$.

$$A'(x) = \underline{\hspace{10em}} \quad (6 \text{ pts})$$



- c) Let x be the value at which $A'(x) = 0$. Give the value of x (including its units). Give the value of $A(x)$ (including its units). Indicate if this is a maximum or minimum for $A(x)$.

$$x = \underline{\hspace{10em}} \quad (3 \text{ pts})$$

$$A(x) = \underline{\hspace{10em}} \quad (3 \text{ pts})$$

$$\text{Max/Min: } \underline{\hspace{10em}} \quad (3 \text{ pts})$$

page total: (27 pts)