Integration by Parts Worksheet

Name _____

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In this worksheet we'll study the technique of integration by parts. Recall that the basic form looks like this:

$$\int uv' = uv - \int vu'$$

1. First a warm-up problem. Consider the integral $\int x \sin(3x) \ dx$. Let $u = x$ and let $v' = \sin(3x)dx$. Compute u' by differentiating and v by integrating, and use the basic formula to compute the integral. Don't forget the arbitrary constant!

2. Suppose we try to integrate $1/x$ by parts, taking $u = 1/x$ and $v' = dx$. We have $u' = (-1/x^2)dx$ and $v = x$, so

$$\int \frac{1}{x} \ dx = \frac{1}{x} \cdot x - \int x \cdot \frac{-1}{x^2} \ dx$$

$$= 1 + \int \frac{1}{x} \ dx.$$

Canceling the integral from both sides, we get the disconcerting result that 0=1. *What went wrong?* What happens if we replace the indefinite integrals by definite integrals, that is, if we try to calculate $\int_a^b \frac{1}{x} \ dx$ by this method?