Partial Fractions Worksheet

Name _____

February 12, 2007

1. (a) Consider the rational function $f(x) = \frac{2x^3-4x^2-5x+3}{x^2-2x-3}$. Use long division to get a quotient and a remainder, then write $f(x) =$ quotient + (remainder/divisor).

   (b) Now consider the expression $\frac{x+3}{x^2-2x-3}$. Factor the denominator into two linear terms.

   (c) We wish to write $\frac{x+3}{(x-3)(x+1)}$ as a sum $\frac{A}{x-3} + \frac{B}{x+1}$. Let's find A and B. Set the two expressions equal and clear denominators (that is, multiply through by $(x-3)(x+1)$ and cancel $(x-3)$'s and $(x+1)$'s as much as possible. Solve for A and B. Check your work by adding the two fractions together.

   (d) Now use the results of the prior problems to compute $\int \frac{2x^3-4x^2-5x+3}{x^2-2x-3}\,dx$.

   (e) Some of the terms in the answer to problem 1d involve logarithms. Combine those terms into a single terms of the form ln(some function of x).

2. Next, consider the function $f(x) = \frac{3x+1}{x(x+1)^2}$. The problem here is that one of the linear terms in the denominator is squared. Partial fraction theory says that the best we can do is to get this one in the form $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$.

   (a) Find A, B, and C. Remember to first set the two expressions equal and clear denominators.

   (b) Now that you've found A, B, and C, compute $\int \frac{3x+1}{x(x+1)^2}\,dx$.