MA2160: Calculus with Technology II
FALL 2006 FINAL EXAM

Non-Calculator Section

NAME: Key (PLEASE PRINT) MTU ID:

CIRCLE YOUR SECTION NUMBER AND INSTRUCTOR'S NAME

Section	Instructor	Time
01	T. King	10:05-10:55am
02	S. Tao	12:05-12:55pm
03	L. Erlbach	3:05-3:55pm
04	L. Erlbach	4:05-4:55pm
05	J. Hilgers	8:05-8:55pm
06	R. Kolkka	11:05-11:55am
08	R. Kolkka	8:05-8:55am
09	T. King	9:05-9:55am
10	E. Westlund	8:05-8:55am

Page	Score
1	/ 10 pts
2	/ 10 pts
3	/ 11 pts
4	/ 15 pts
5	/ 6 pts
6	/ 10 pts
7	/ 4 pts
8	/ 11 pts
9	/ 6 pts
SubTotal	/ 83 pts
SubTotal of Calculator Section	/ 17 pts
TOTAL	/ 100 pts

Show sufficient work to justify all answers. Calculators are NOT ALLOWED on this section of the exam. WORK THIS NON-CALCULATOR PART FIRST. It must be turned in BEFORE you use your calculator on Part II.

Non-Calculator Section.

1. Evaluate $\int_2^\infty \frac{1}{(x-1)^2}\,dx$.

$$\int_2^\infty \frac{1}{(x-1)^2}\,dx = \lim_{a\to\infty} \int_2^a \frac{1}{(x-1)^2}\,dx$$

$$= \lim_{a\to\infty} \int_2^a (x-1)^{-2}\,dx$$

$$= \lim_{a\to\infty} -(x-1)^{-1}\Big|_2^a$$

$$= \lim_{a\to\infty}\left(\frac{1}{1-a}\right) - \frac{1}{1-2}$$

$$= 0 + 1$$

$$= 1$$

ANSWER: 1 [5 pts]

2. Evaluate $\int \frac{\ln(x)}{x}\,dx$.

$$\int \frac{\ln x}{x}dx \overset{u=\ln x}{=\!=} \int u\,du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C$$

ANSWER: $\frac{(\ln x)^2}{2} + C$ [5 pts]

3. Evaluate $\int te^{5t}\, dt$.

$u = t$, $v' = e^{5t}$

$u' = 1$ $\quad v = \frac{1}{5}e^{5t}$

$$\int te^{5t}dt = uv - \int u'v\,dt$$
$$= \frac{t}{5}e^{5t} - \int \frac{1}{5}e^{5t}dt$$
$$= \frac{t}{5}e^{5t} - \frac{1}{25}e^{5t} + C$$

ANSWER: $\frac{t}{5}e^{5t} - \frac{1}{25}e^{5t} + C$ [5 pts]

4. Evaluate $\int \frac{x}{x^2 - 3x + 2}\, dx$.

$$\frac{x}{x^2-3x+2} = \frac{A}{x-2} + \frac{B}{x-1}$$

$$x = A(x-1) + B(x-2)$$

$$x = Ax + Bx - A - 2B$$

$$x = (A+B)x + (-A-2B)$$

Thus $\left.\begin{array}{l} A+B=1 \\ -A-2B=0 \end{array}\right\} \Rightarrow \begin{array}{l} A=2 \\ B=-1 \end{array}$

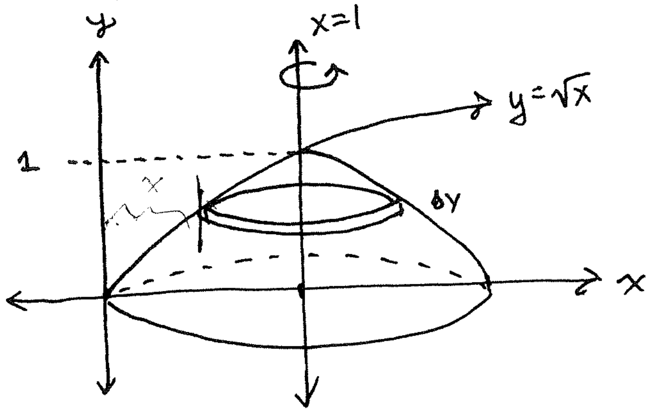
$$\int \frac{x}{x^2-3x+2}dx = \int \left(\frac{2}{x-2} + \frac{-1}{x-1}\right)dx$$
$$= \int \frac{2}{x-2}dx - \int \frac{1}{x-1}dx$$
$$= 2\ln|x-2| - \ln|x-1| + C$$

or $\ln \frac{(x-2)^2}{|x-1|} + C$

ANSWER: $2\ln|x-2| - \ln|x-1| + C$ [5 pts]

5. Consider the region R bounded by the curves $y = \sqrt{x}$, $x = 1$, and $y = 0$.

(a) Neatly sketch the volume of revolution obtained by rotating the region R about the line $x = 1$. (Clearly label your graph.) [2 pts]



(b) Find the resulting volume.

Volume of an arbitrary slice is

$$\pi(1-x)^2\,\Delta y = \pi(1-y^2)^2\,\Delta y$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\pi(1-y^2)^2\,\Delta y = \pi\int_0^1 1-2y^2+y^4\,dy = \pi\left(y-\frac{2}{3}y^3+\frac{y^5}{5}\right)\Big|_0^1 = \pi\left(1-\frac{2}{3}+\frac{1}{5}\right)$$

$$= \pi\left(\frac{1}{3}+\frac{1}{5}\right) = \frac{8\pi}{15}$$

VOLUME = $8\pi/15$ [4 pts]

6. A rod of length 1 meter has a mass density $\delta(x) = 1 + kx^2$ grams/meter, where k is a positive constant. The rod is lying along the positive x-axis, with the left end at the origin. Find the center of mass $\bar{x}$ as a function of k.

$$\bar{x} = \frac{\int_0^1 x\,\delta(x)\,dx}{\int_0^1 \delta(x)\,dx} = \frac{\int_0^1 x(1+kx^2)\,dx}{\int_0^1 (1+kx^2)\,dx} = \frac{\frac{x^2}{2}+\frac{kx^4}{4}\Big|_0^1}{x+\frac{kx^3}{3}\Big|_0^1} = \frac{\frac{1}{2}+\frac{k}{4}}{1+\frac{k}{3}} = \frac{\frac{2+k}{4}}{\frac{3+k}{3}}$$

$$\frac{\frac{1}{2}\left(1+\frac{k}{2}\right)}{\left(1+\frac{k}{3}\right)} =$$

$$\bar{x} = \frac{3(2+k)}{4(3+k)}$$

ANSWER: $\bar{x} = \frac{3(2+k)}{4(3+k)}$ [5 pts]

$4\pi\left(\frac{1}{3}-\frac{1}{5}\right)$

7. Find the sum of the infinite series:

$$\frac{2}{3} - \frac{1}{6} + \frac{1}{24} - \frac{1}{96} + \frac{1}{384} - \frac{1}{1536} + \cdots$$

Express your answer as an **integer or a simple fraction** (containing no square roots, powers, factorials, logs, etc.)

$$\frac{2}{3}\left[1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots\right] = \frac{2}{3}\left[\frac{1}{1+\frac{1}{4}}\right] = \frac{8}{15}$$

(2) (2) (1)

ANSWER:____________________[5 pts]

8. Write the 2^{nd} degree Taylor polynomial for the function $f(x) = \frac{1}{x^2}$ centered about 2. Write the coefficients as either **integers or simple fractions** (containing no square roots, powers, factorials, logs, etc.)

-1 for higher degree terms

$$\left.\begin{aligned} f(x) &= x^{-2} \\ f'(x) &= -2x^{-3} \\ f''(x) &= 6x^{-4} \end{aligned}\right\} \quad \begin{aligned} f(2) &= \frac{1}{4} \\ f'(2) &= -\frac{1}{4} \\ f''(2) &= \frac{3}{8} \end{aligned}$$

$$\frac{1}{4} - \frac{1}{4}(x-2) + \frac{3}{16}(x-2)^2$$

(1) (1) (1) (1) (1)

ANSWER:____________________[5 pts]

9. Write only the first 5 nonzero terms in a Taylor series for the function f centered about 0 where

$$f(x) = 8 + \sin(x^2) + e^{-(x^2)}.$$

$$8 = 8$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!}$$

$$e^{-(x^2)} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!} + \frac{x^{12}}{6!}$$

-1 for each x^n that doesn't belong,

e.g.: $x, x^2, x^3, x^5, x^7, x^9, x^{10}$

$$9 + \frac{x^4}{2} - \frac{x^6}{3} + \frac{x^8}{24} + \frac{x^{12}}{720}$$

ANSWER:____________________[5 pts]

(1) (1) (1) (1) (1)

10. In a certain chemical reaction, ($A \to B$), the rate of the reaction is observed to be proportional (with proportionality constant k) to the amount of reactant A present.

(a) Write a differential equation satisfied by A, the amount of reactant A, as a function of time t.

$$\frac{dA}{dt} = kA$$

ANSWER: $\frac{dA}{dt} = kt$ [2 pts]

(b) Find the general solution, expressing A as an explicit function of time t.

$$\ln A = kt + c$$

$$A = Ce^{kt}$$

ANSWER: $A = Ce^{kt}$ [2 pts]

(c) Find the particular (specific) solution for the initial condition $A(0) = A_0$.

$A(0) = A_0$

$$A = Ce^{kt}$$

$$A_0 = Ce^{0} \Rightarrow C = A_0$$

ANSWER: $A = A_0 e^{kt}$ [1 pts]

(d) Evaluate the proportionality constant k if $A(1) = 0.5A_0$.

$$\frac{A}{A_0} = 0.5 = e^{k}$$

$$k = \ln(0.5)$$

ANSWER: $\ln(0.5) = k$ [1 pts]

11. For what values of ω does $y = \sin(\omega t)$ satisfy the differential equation below?

$$\frac{d^2y}{dt^2} + 4y = 0$$

$y = \sin \omega t$

$y' = \omega \cos \omega t$

$y'' = -\omega^2 \sin \omega t$

3 $\quad -\omega^2 \sin \omega t + 4 \sin \omega t = 0$

$(4 - \omega^2) \sin \omega t = 0$

2 $\quad \omega = \pm 2, 0$

trivial

ANSWER: $\pm 2, 0$ [5 pts]

12. Solve the following initial value problem:

$$\frac{dy}{dx} = -\frac{x}{y}, \quad y(0) = 4$$

1 $\quad y\,dy = -x\,dx$

2 $\quad \frac{y^2}{2} = -\frac{x^2}{2} + c$

$y^2 = -x^2 + c_1$

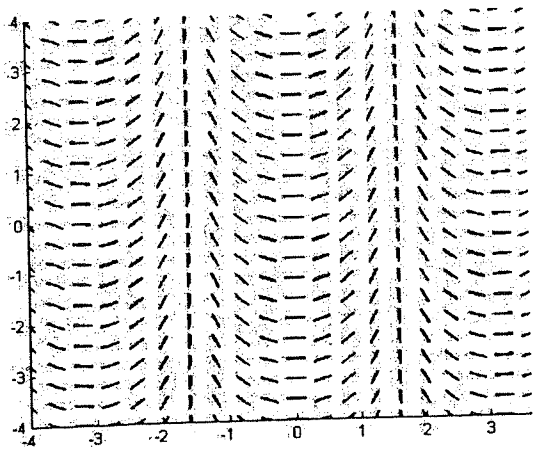
2 $\quad y(0) = 4 \Rightarrow 16 = (0)^2 + c_1 \qquad c_1 = 16$

ANSWER: $y = \sqrt{16 - x^2}$ [5 pts]

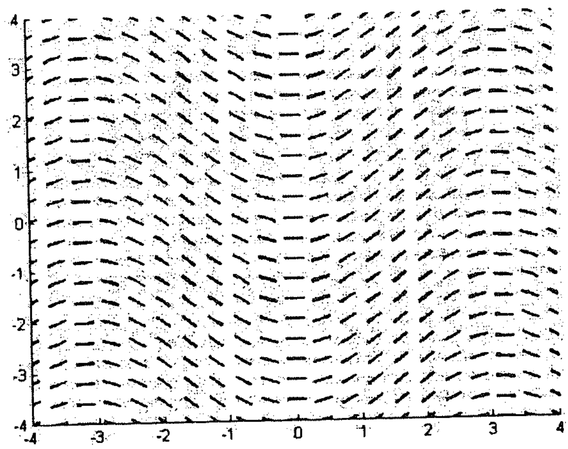
$y^2 = 16 - x^2 \qquad y = \sqrt{16 - x^2}$

13. Match the given slope fields with their corresponding differential equation. [4 pts]

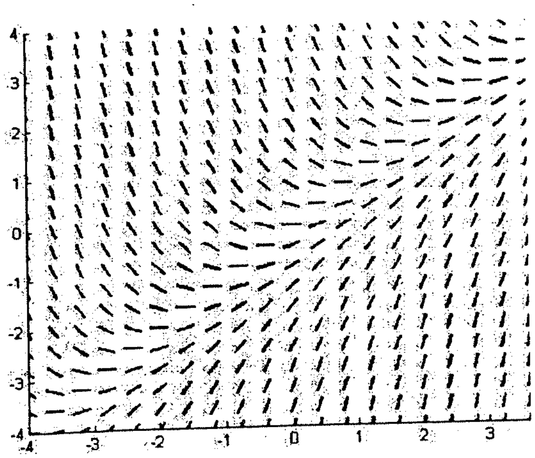
(*a*) $y' = y - 2$ (*b*) $y' = \tan(x)$ (*c*) $y' = \sin(x)$ (*d*) $y' = x + y$



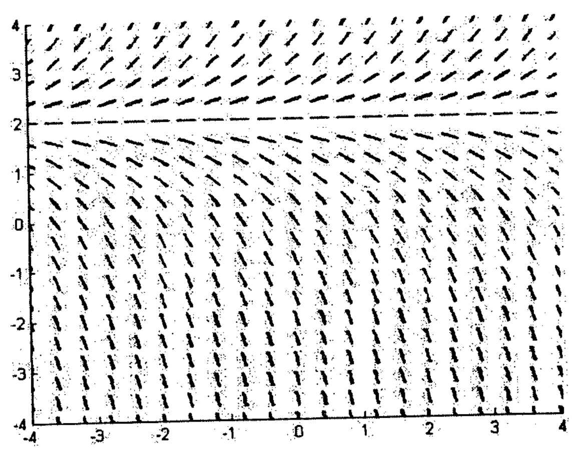
b



c



d



a

14. Consider the differential equation $\frac{dy}{dt} = y - 2$ where $y(0) = 1$. Use Euler's Method with step-size $\Delta t = 1$ to estimate $y(5)$. Show your work in the table below. [6 pts]

t	y	dy/dt	$\Delta t \cdot \frac{dy}{dt}$
0	1	-1	-1
1	0	-2	-2
2	-2	-4	-4
3	-6	-8	-8
4	-14	-16	-16
5	-30		

15. Give a unit vector perpendicular to both $\vec{u} = 3\hat{\imath} - 3\hat{\jmath}$ and $\vec{v} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

$$\begin{vmatrix} i & j & k & i & j \\ 3 & -3 & 0 & 3 & -3 \\ 1 & 2 & 1 & 1 & 2 \end{vmatrix}$$

$$= -3\hat{\imath} + 0\hat{\jmath} + 6\hat{k} - 3\hat{\jmath} + 0\hat{\imath} + 3\hat{k}$$

$$\boxed{\hat{u} \times \hat{v} = -3\hat{\imath} - 3\hat{\jmath} + 9\hat{k}}$$

The unit vector is $\frac{\hat{u} \times \hat{v}}{\|\hat{u} \times \hat{v}\|} = \frac{-3\hat{\imath} - 3\hat{\jmath} + 9\hat{k}}{\sqrt{3^2 + 3^2 + 9^2}} = -\frac{\hat{i}}{\sqrt{11}} - \frac{\hat{j}}{\sqrt{11}} + \frac{3\hat{k}}{\sqrt{11}}$

ANSWER: $-\frac{\hat{i}}{\sqrt{11}} - \frac{\hat{j}}{\sqrt{11}} + \frac{3\hat{k}}{\sqrt{11}}$ [5 pts]

$\sqrt{99} = \sqrt{9}\sqrt{11} = 3\sqrt{11}$

16. Compute the following:

(a) $2(-\hat{\imath} + 3\hat{\jmath} + \hat{k}) + (6\hat{\imath} - 4\hat{\jmath} + 3\hat{k})$.

$-2\hat{\imath} + 6\hat{\jmath} + 2\hat{k} + 6\hat{\imath} - 4\hat{\jmath} + 3\hat{k}$

ANSWER: $4\hat{\imath} + 2\hat{\jmath} + 5\hat{k} = \langle 4, 2, 5 \rangle$ [2 pts]

(b) If $\vec{u} = 4\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$ and $\vec{v} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, find $\vec{u} \cdot \vec{v}$.

$4(-1) + (-2)(2) + (5)(2) = -4 - 4 + 10 = 2$

ANSWER: 2 [2 pts]

(c) If $\vec{u} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{v} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, find $\vec{u} \times \vec{v}$.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 2 & 1 \\ -2 & 3 & 1 \end{vmatrix} = -\hat{\imath} - 4\hat{\jmath} + 10\hat{k}$$

$$\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} \hat{\imath} - \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} 2 & 2 \\ -2 & 3 \end{vmatrix} \hat{k} =$$

$-\hat{\imath} - 4\hat{\jmath} + 10\hat{k}$

ANSWER: $-\hat{\imath} - 4\hat{\jmath} + 10\hat{k} = \langle -1, -4, 10 \rangle$ [2 pts]

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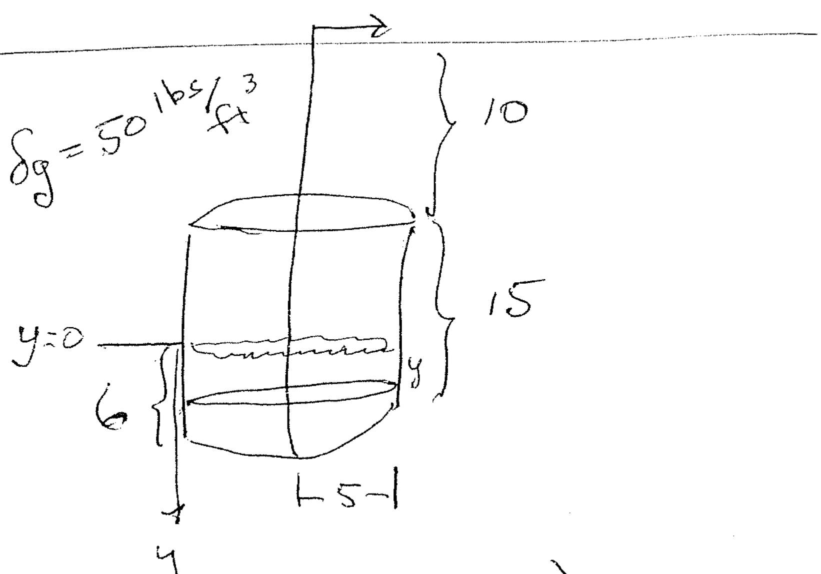
Page	Score
1	/ 6 pts
2	/ 11 pts
SUBTOTAL	/ 17 pts

Show sufficient work to justify all answers. Calculators ARE ALLOWED on this section of the exam. YOU MAY USE YOUR CALCULATOR ON THIS PART AFTER YOU HAVE TURNED IN THE NON-CALCULATOR PART FIRST.

Calculator Section.

17. A fuel oil tank is an upright circular cylinder, buried such that the top is 10 feet beneath ground level. The tank has radius 5 feet and height 15 feet. The depth of the oil in the tank is 6 feet. Calculate the work required to pump all of the oil to the surface. (The density of oil is 50 pounds per cubic foot.)

$\delta g = 50 \text{ lbs}/\text{ft}^3$



$19 \times 6 = 114$

$$dW = \delta_g \, dV\,(y+19)$$
$$= \delta_g \, \pi\, 25\, dy\,(y+19)$$

$$W = \delta_g \pi\, 25 \int_0^6 (y+19)\,dy$$
$$= 50(25)\pi \left[\frac{y^2}{2} + 19y \right]_0^6$$
$$= 1250\pi\,(18+114)$$
$$= \boxed{1250(132)\pi} =$$

WORK: 165000π ft lbs [6 pts]

518362.7878 ft lbs

6

18. Find an equation for the plane through the 3 points $P = (1,0,0)$, $Q = (0,2,0)$, and $R = (0,0,3)$.

$\overrightarrow{PQ} = \langle -1, 2, 0 \rangle$ $\quad \overrightarrow{PR} = \langle -1, 0, 3 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = \langle 6, 3, 2 \rangle$$

5

$$\boxed{6(x-1) + 3y + 2z = 0}$$

ANSWER:______________________________[5 pts]

19. A force of $\vec{F} = \langle 2, 2, -3 \rangle = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ Newtons results in a displacement of $\vec{D} = \langle -1, 4, 1 \rangle = -\hat{\imath} + 4\hat{\jmath} + \hat{k}$ meters.

(a) What is the total work done?

$$W = \vec{F} \cdot \vec{D} = \langle 2, 2, -3 \rangle \cdot \langle -1, 4, 1 \rangle$$

$$= 2(-1) + 2(4) - 3(1) = \boxed{3 \text{ Joules}}$$

3

WORK:______________________________[3 pts]

(b) What is the component of the force parallel to $\vec{D}$?

$$\overrightarrow{\text{Comp}}_{\vec{D}} \vec{F} = \frac{\vec{F} \cdot \vec{D}}{\|\vec{D}\|^2} \vec{D} = \frac{3}{18} \langle -1, 4, 1 \rangle$$

$$= \frac{1}{6} \langle -1, 4, 1 \rangle$$

3

ANSWER: $= \langle -\frac{1}{6}, \frac{2}{3}, \frac{1}{6} \rangle$ ______________________________[3 pts]

11