More Practice Problems

MA2160

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1. Graph three vectors $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{c}$, such that
   (a) $\overrightarrow{a} = 2\overrightarrow{b}$ and $\overrightarrow{c} = 3\overrightarrow{b}$
   (b) $\overrightarrow{a} = \overrightarrow{b} + \overrightarrow{c}$, where $\overrightarrow{b}$ and $\overrightarrow{c}$ are not parallel
   (c) $\overrightarrow{a} = \overrightarrow{b} - \overrightarrow{c}$, where $\overrightarrow{b}$ and $\overrightarrow{c}$ are not parallel
   (d) $\overrightarrow{a} = \overrightarrow{b} + \overrightarrow{c}$ and $\overrightarrow{b} \cdot \overrightarrow{c} = 0$
2. Consider the vectors

$$\begin{aligned}
\overrightarrow{a} &= 3\hat{\imath} + \hat{\jmath} - 2\hat{k} \\
\overrightarrow{b} &= -2\hat{\imath} - 3\hat{\jmath} - 2\hat{k} \\
\overrightarrow{c} &= -6\hat{\imath} - 2\hat{\jmath} + 4\hat{k} \\
\overrightarrow{d} &= -\hat{\imath} + 2\hat{\jmath} \\
\overrightarrow{e} &= \hat{\imath} - \hat{\jmath} + \hat{k} \\
\overrightarrow{f} &= -2\hat{\imath} - 1\hat{\jmath} + 3\hat{k} \\
\overrightarrow{g} &= 3\hat{\imath} - \hat{\jmath} - 2\hat{k}
\end{aligned}$$

   (a) $\overrightarrow{a} + \overrightarrow{b} =$
   (b) $\overrightarrow{a} \cdot \overrightarrow{b} =$
   (c) $\overrightarrow{a} \times \overrightarrow{b} =$
   (d) $3\overrightarrow{c} \cdot (-\overrightarrow{d}) =$
   (e) $3\overrightarrow{e} - 2\overrightarrow{f} + \overrightarrow{c} =$
   (f) $||\overrightarrow{c}|| =$
   (g) $||12\overrightarrow{f}|| =$
   (h) $\overrightarrow{a} + \overrightarrow{c} =$
   (i) $\overrightarrow{b} - \overrightarrow{d} =$
   (j) $-(\overrightarrow{b} \times \overrightarrow{a}) =$
   (k) Are $\overrightarrow{a}$ and $\overrightarrow{f}$ parallel to each other?
   (l) Are $\overrightarrow{a}$ and $\overrightarrow{c}$ parallel to each other?
   (m) Is $\overrightarrow{b}$ perpendicular to $\overrightarrow{f}$?
   (n) Find all vectors that are perpendicular to $\overrightarrow{e}$.
   (o) Find a vector of length 10 in direction of $\overrightarrow{c}$.

(p) For what values of λ are $\overrightarrow{c}$ and $\overrightarrow{g}$ parallel to each other? Are there any values of λ for which they are perpendicular?

(q) What is the length of $\overrightarrow{e}$?

(r) Find the angle between $\overrightarrow{d}$ and the positive x-axis.

(s) Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

(t) Find the equation of the plane perpendicular to $\overrightarrow{e}$ and passing through the point $(1, 2, 1)$.

(u) Find the equation of the plane perpendicular to $\overrightarrow{b}$ and passing through the point $(2, 2, 0)$.

(v) Find the area of the parallelogram with edges $\overrightarrow{a}$ and $\overrightarrow{d}$.

(w) Find the volume of the parallelpiped with edges $\overrightarrow{a}$, $\overrightarrow{b}$ and $\overrightarrow{e}$.

(x) Find a unit vector perpendicular to $\overrightarrow{a}$ and $\overrightarrow{e}$.

3. Consider the points $P = (1, 2, 3)$, $Q = (3, 1, 4)$ and $R = (2, 5, 6)$.

   (a) Find the vector $\overrightarrow{PQ}$.

   (b) Compute the distance between P and Q.

   (c) Find the cosine of the angle PQR at vertex Q.

   (d) Find the angle between PQ and QR.

   (e) Compute the area of the triangle PQR.

   (f) Find the length of all sides of the triangle PQR.

   (g) Compute the distance from R to the line through P and Q.

   (h) Find a unit vector perpendicular to a plane containing P, Q and R.

   (i) Find an equation of the plane passing through the three points P, Q and R.

4. Two forces, represented by the vecotrs $\overrightarrow{F_1} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\overrightarrow{F_2} = 2\hat{\imath} + 3\hat{k}$ are applied on an object. Give a vector representing the force that needs to be applied to the object if it is to remain stationary.

5. A 100-meter dash is run on a track in direction of the vector $\overrightarrow{v} = 3\hat{\imath} + 4\hat{\jmath}$. The wind velocity is $\overrightarrow{w} = 2\hat{\imath} + 6\hat{\jmath}$. The rules say that a legal wind speed measured in the direction of the dash must not exceed 5 km/hr. Will the race results be disqualified due to an illegal wind? Justify your answer.

6. Suppose two vectors $\overrightarrow{x}$ and $\overrightarrow{y}$ satisfy $\overrightarrow{p} \cdot \overrightarrow{x} = 5$ and $\overrightarrow{p} \cdot \overrightarrow{y} = 5$ respectively. Show that $\overrightarrow{p}$ and $\overrightarrow{q} = \overrightarrow{x} - \overrightarrow{y}$ are orthogonal.

7. Find a vector $\overrightarrow{v}$ parallel to the line of intersection of the two planes $x - 2(y-1) + 3(z+2) = 0$ and $2(x-1) + (y+1) - 2(z-1) = 0$.

8. Compute the following integrals:

   (a) $\int e^{-x}dx$
   (b) $\int xe^{-x^2}dx$
   (c) $\int 4xe^{-4x^2}dx$
   (d) $\int \frac{5e^{2x}}{1+e^{2x}}dx$
   (e) $\int \frac{e^x}{(e^x+a)^2}dx$
   (f) $\int \sin xe^{2\cos x}dx$
   (g) $\int \sin\theta(\cos\theta+2)^3d\theta$
   (h) $\int \frac{e^t+1}{e^t+t}dt$
   (i) $\int \frac{1+2e^{2x}}{\sqrt{x+e^{2x}}}dx$
   (j) $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$
   (k) $\int e^{-A\cos\theta}A\sin\theta d\theta$
   (l) $\int \frac{5x-2}{\sqrt{x+1}}dx$

9. Compute the following integrals:

   (a) $\int xe^{-x}dx$
   (b) $\int t\cos tdt$
   (c) $\int \ln xdx$
   (d) $\int x^3\ln xdx$
   (e) $\int 2x^2\sin xdx$
   (f) $\int 2\cos^2 tdt$
   (g) $\int 2(\ln x)^2dx$
   (h) $\int \frac{t+8}{\sqrt{4-t}}dt$
   (i) $\int x^5\cos(x^3)dx$
   (j) $\int \ln(1+t)dt$
   (k) $\int \arcsin tdt$

10. Let f be twice differentiable with $f(1)=2$, $f(2)=5$, $f'(1)=1$ and $f'(2)=3$. Evaluate the integral $\int_1^2 xf''(x)dx$.

11. Compute the following integrals:

   (a) $\int \frac{1}{(x-2)(x-3)}dx$
   (b) $\int \frac{x+1}{x^2+2x}dx$
   (c) $\int \frac{8x-4x^2}{(x-1)^2(x+3)}dx$
   (d) $\int \frac{x^2-2x-\frac{1}{3}}{(x^2+1)(x-3)}dx$
   (e) $\int \frac{x^4-2x^3+x-1}{x^2-2x}dx$
   (f) $\int \frac{1}{x^2-4}dx$
   (g) $\int \frac{3}{4x-x^2}dx$
   (h) $\int \frac{x^5-2x^4+x^3-3x-1}{x^2-2x+1}dx$