4.2 #19

$y'' - 3y' + 2y = 5e^{3x}$ $\quad y_1 = e^x$

$y = e^x u$

$y' = e^x u + e^x u'$

$y'' = e^x u'' + 2e^x u' + e^x u$

Note:
$y_1 = e^x$
$y_2 = e^{2x}$
and $y_p = \frac{5}{2}e^{3x}$

y'' $\quad e^x u'' + 2e^x u' + e^x u$

$-3y'$ $\quad -3e^x u' - 3e^x u$

$+2y$ $\quad +2e^x u$

$5e^{3x} = e^x u'' - e^x u' \qquad w = u'$

$e^x w' - e^x w = 5e^{3x}$ $\quad$ 1st order lin

$w' - 1w = 5e^{2x} \qquad N(x) = e^{-x}$

$(e^{-x} w)' = 5e^{2x} e^{-x} = 5e^x$

$we^{-x} = 5e^x + C_7 \qquad u' = w = 5e^{2x} + C_7 e^x$

$u = \frac{5}{2} e^{2x} + C_7 e^x + C_1$

$y = e^x\left(\frac{5}{2} e^{2x} + C_7 e^x + C_1\right) = \frac{5}{2} e^{3x} + C_2 e^{2x} + C_1 e^x$