

4.4  
#20

$$y'' + 2y' - 24y = 16 - (x+2)e^{4x}$$

$$m^2 + 2m - 24 = 0$$

$$m = -6 \text{ and } m = 4$$

$$y_c = c_1 e^{-6x} + c_2 e^{4x}$$

$$y_{p1} = A_1 \quad y_{p2} = x(Ax + B)e^{4x}$$

$$y_{p1}' = 0$$

$$y_{p1}'' = 0$$

$$0 + 2(0) - 24A_1 = 16$$

$$A_1 = -\frac{16}{24} = -\frac{2}{3}$$

$$y_{p2} = Ax^2 e^{4x} + Bx e^{4x}$$

$$y_{p2}' = 2Ax e^{4x} + 4Ax^2 e^{4x} + B e^{4x} + 4Bx e^{4x}$$

$$y_{p2}' = 4Ax^2 e^{4x} + (2A + 4B)x e^{4x} + B e^{4x}$$

$$y_{p2}'' = 16Ax^2 e^{4x} + 8Ax e^{4x}$$

$$+ 4(2A + 4B)x e^{4x} + (2A + 4B)e^{4x}$$

$$y_{p2}'' = 16Ax^2 e^{4x} + (16A + 16B)x e^{4x} + (2A + 8B)e^{4x}$$

4.4 #20 continued

$$\begin{aligned} Y_p'' & 16A x^2 e^{4x} + (16A + 16B) x e^{4x} + (2A + 8B) e^{4x} \\ + 2Y_p' & + 8A x^2 e^{4x} + 2(2A + 4B) x e^{4x} + 2B e^{4x} \\ - 24Y_p & - 24A x^2 e^{4x} - 24B x e^{4x} \end{aligned}$$

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$$-(x+2)e^{4x} \quad 0(A) x^2 e^{4x} + (20A - 0B) x e^{4x} + (2A + 10B) e^{4x}$$

So  $x^2 e^{4x}$  term vanishes!

$$x e^{4x} \text{ term balances if } 20A = -1$$

$$A = -\frac{1}{20}$$

$$e^{4x} \text{ term balances if } 2A + 10B = -2 \quad B = \frac{-14}{20}$$

$$Y = Y_c + Y_{p1} + Y_{p2}$$