4.6 #19 $y(0)=1$, $y'(0)=0$

$4y'' - y = xe^{x/2}$ $\quad y'' - \frac{1}{4}y = \frac{1}{4}xe^{x/2}$

$4m^2 - 1 = 0 \quad m = +\frac{1}{2}$ or $-\frac{1}{2}$

$y_c = c_1 y_1 + c_2 y_2 \quad y_1 = e^{x/2}$, $y_2 = e^{-x/2}$

$u_1' e^{x/2} + u_2' e^{-x/2} = 0$

$u_1' \frac{1}{2} e^{x/2} + u_2' \left(-\frac{1}{2} e^{-x/2}\right) = \frac{1}{4} x e^{x/2}$

(x2) $\rightarrow u_1' e^{x/2} - u_2' e^{-x/2} = \frac{1}{2} x e^{x/2}$

(add) $2u_1' e^{x/2} + 0 = \frac{1}{2} x e^{x/2}$

$u_1' = x\frac{1}{4}$ ~~$u_2 = -e^{x/2} u_1' / e^{-x/2} = -\frac{1}{4} e^{3x/2}$~~

$u_2' = -e^{x/2} u_1' / e^{-x/2} = -x/4\, e^x$

$u = x$, $dv = e^x dx$

$\boxed{u_1 = x^2/8 + c_1}$ $\quad u_2 = -\frac{1}{4}\int xe^x dx$

$u_2 = -\frac{1}{4}\left(xe^x - \int e^x dx\right)$

$\boxed{u_2 = -\frac{1}{4}(xe^x - e^x) + c_2}$

$$y = \left(\frac{x^2}{2} + C_1\right)e^{x/2} + \left(-\frac{1}{4}(xe^x - e^x) + C_2\right)e^{-x/2}$$

$y(0)=1$ $\quad 1 = \left(C_1\right)e^0 + \left(-\frac{1}{4}(0-1) + C_2\right)e^0$

$$\boxed{C_1 + C_2 = 1 - \frac{1}{4} = \frac{3}{4}}$$

$$y' = \left(\frac{2x}{2}\right)e^{x/2} + \left(\frac{x^2}{2} + C_1\right)e^{x/2}\cdot\frac{1}{2}$$
$$-\frac{1}{4}\left(xe^x - e^x - e^x\right)e^{-x/2} + -\frac{1}{2}\left(-\frac{1}{4}(xe^x - e^x) + C_2\right)e^{-x/2}$$

$y'(0)=0$ $\quad 0 = 0 + C_1\frac{1}{2} - \frac{1}{4}(-2) - \frac{1}{2}\left(-\frac{1}{4} + C_2\right)$

$$\boxed{0 = \frac{C_1}{2} + \frac{1}{2} + \frac{1}{8} - \frac{C_2}{2}}$$

$$C_1 + C_2 = \frac{3}{4}$$
$$C_1 - C_2 = \left(-\frac{1}{2} - \frac{1}{8}\right)2 = -\frac{5}{4}$$

add $\quad 2C_1 = \frac{3}{4} - \frac{5}{4} = -\frac{1}{2} \qquad \boxed{C_1 = -\frac{1}{4}}$

$$-\frac{1}{4} + C_2 = \frac{3}{4} \qquad \boxed{C_2 = 1}$$

or $\quad \left[\begin{array}{cc|c} 1 & 1 & \frac{3}{4} \\ 1 & -1 & -\frac{5}{4} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & 1 \end{array}\right]$