4.6 #3 4.6 intro

$y'' + y = \sin(x)$ $y_c = c_1 y_1 + c_2 y_2$

$y_1 = \sin(x)$ $y_2 = \cos(x)$

$\sin(x)\, u_1' + \cos(x)\, u_2' = 0$
$\cos(x)\, u_1' - \sin(x)\, u_2' = \sin(x)$

$\sin^2(x)\, u_1' + \sin(x)\cos(x)\, u_2' = 0$
$\cos^2(x)\, u_1' - \sin(x)\cos(x)\, u_2' = \sin(x)\cos(x)$

$u_1' = \sin(x)\cos(x)$

$u_2' = \dfrac{-\sin(x)\, u_1'}{\cos(x)} = -\sin^2(x)$

$u_1 = \int \sin(x)\cos(x)\,dx = \int v\,dv = v^2/2$

$v = \sin(x)$
$dv = \cos(x)\,dx$

$= \sin^2(x)/2 + C_1$

$u_2 = -\left(x/2 - \frac{1}{4}\sin(2x)\right) + C_2$

$y = u_1 y_1 + u_2 y_2$