8.2 # 39

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\lambda = \alpha \pm \beta I$

$\lambda_1 = 0 \qquad \lambda_2 = 0 + (1)I \qquad \alpha = 0$

$\lambda_3 = 0 - (1)I \qquad \beta = 1$

$$K_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} I$$

$$B_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad B_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$x = C_1 e^{0t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 e^{\alpha t}\Big(\cos(\beta t) B_1 - \sin(\beta t) B_2\Big)$$

$$+ C_3 e^{\alpha t}\Big(\sin(\beta t) B_1 + \cos(\beta t) B_2\Big)$$

"$K_3 = B_1 - B_2 I$"

extra IC $\qquad x(0) = \{1, 2, 3\}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 3 \end{array}\right]$$

$$C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + C_3 \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$