

EX2/

Solve $X' = AX + \begin{pmatrix} 3 \cos(2t) \\ \sin(2t) \end{pmatrix}$

$$A = \begin{bmatrix} 1 & -7 \\ 1 & 9 \end{bmatrix}$$

$$X_c = c_1 e^{2t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} -7 \\ 1 \end{bmatrix}$$

GUESS

$$X_p = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cos(2t) + \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \sin(2t)$$

$$X_p' = -2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \sin(2t) + 2 \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \cos(2t)$$

$$\begin{pmatrix} -2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \sin(2t) \\ + 2 \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \cos(2t) \end{pmatrix} = \begin{pmatrix} A \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} \sin(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \\ A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cos(2t) + \begin{pmatrix} 3 \\ 0 \end{pmatrix} \cos(2t) \end{pmatrix}$$

$$-2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = A \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$2 \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$-2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - A \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$-A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + 2 \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Ex2

$$\left[\begin{array}{cc|c} -2 & 0 & 1 \\ 0 & -2 & -A \\ \hline -A & 2 & 0 \\ & 0 & 2 \end{array} \right] \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} -2 & 0 & -1 & 7 & 0 \\ 0 & -2 & -1 & -9 & 1 \\ -1 & 7 & 2 & 0 & 3 \\ -1 & -9 & 0 & 2 & 0 \end{array} \right] \sim \left[\begin{array}{cccc|c} & & & & -73/68 \\ & & & & 5/68 \\ & & & & 12/17 \\ & & & & -7/34 \end{array} \right] \text{Id}_4$$

$$a_1 = -73/68 \quad a_2 = 5/68$$

$$a_3 = 12/17 \quad a_4 = -7/34$$

$$x = x_c + x_p$$