

Ex 3

Solve  $X' = AX + \begin{pmatrix} e^{3t} \\ 2e^{3t} \end{pmatrix}$

$$A = \begin{bmatrix} 1 & -7 \\ 1 & 9 \end{bmatrix}$$

$$X_c = c_1 e^{2t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} -7 \\ 1 \end{bmatrix}$$

*guess*

$$X_p = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t}$$

$$X_p' = 3 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t}$$

$$3 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t} = A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{3t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}$$

$$3 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$(3I_2 - A) \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\left[ \begin{array}{cc|c} 3-1 & 7 & 1 \\ -1 & 3-9 & 2 \end{array} \right] = \left[ \begin{array}{cc|c} 2 & 7 & 1 \\ -1 & -6 & 2 \end{array} \right]$$

$$\boxed{\begin{matrix} a_1 = 4 \\ a_2 = -1 \end{matrix}} \quad X = X_c + X_p \quad \left[ \begin{array}{cc|c} 1 & 6 & 4 \\ 0 & 1 & -1 \end{array} \right]$$