

EX4/

Solve $X' = AX + \begin{pmatrix} 17t \\ 27t \end{pmatrix}$

$$A = \begin{bmatrix} 1 & -7 \\ 1 & 9 \end{bmatrix}$$

$$X_c = c_1 e^{2t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} -7 \\ 1 \end{bmatrix} \quad \boxed{X = X_c + X_p}$$

GUESS

$$X_p = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} t$$

$$X_p' = \begin{pmatrix} a_3 \\ a_4 \end{pmatrix}$$

$$\begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + A \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} t + \begin{pmatrix} 17 \\ 27 \end{pmatrix} t$$

$$A \begin{pmatrix} a_3 \\ a_4 \end{pmatrix} + \begin{pmatrix} 17 \\ 27 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{"t EQ"}$$

$$\begin{pmatrix} a_3 \\ a_4 \end{pmatrix} = A \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \text{"const EQ"}$$

$$t\text{-EQ} \quad \left[\begin{array}{cc|c} 1 & -7 & -17 \\ 1 & 9 & -27 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 0 & -\frac{171}{8} \\ 0 & 1 & -\frac{5}{8} \end{array} \right] \quad \begin{cases} a_3 = -171/8 \\ a_4 = -5/8 \end{cases}$$

$$\text{const-EQ} \quad \left[\begin{array}{cc|c} 1 & -7 & -171/8 \\ 1 & 9 & -5/8 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 0 & -727/64 \\ 0 & 1 & 23/64 \end{array} \right] \quad \begin{cases} a_1 = -727/64 \\ a_2 = 23/64 \end{cases}$$