§ 2.3 $y(0)=0$

#40 $(1+x^2)\,dy/dx + 2xy = \begin{cases} x & 0 \le x < 1 \\ -x & x \ge 1 \end{cases}$

$(1+x^2)\,dy/dx + 2xy = x$ $y(0)=0$

$dy/dx + \frac{2x}{1+x^2}\,y = \frac{x}{1+x^2}$

$\mu(x) = e^{\int \frac{2x}{1+x^2}\,dx}$ $u = 1+x^2$, $du = 2x\,dx$

$\mu(x) = e^{\ln(1+x^2)} = 1+x^2$

$(1+x^2)\,dy/dx + 2xy = x$

$\frac{d}{dx}\left[(1+x^2)y\right] = x$

$(1+x^2)y = \frac{x^2}{2} + C_1$ $y(0)=0$

$(1+0)\,0 = 0^2/2 + C_1$ $\boxed{C_1 = 0}$

$y(x) = \begin{cases} \frac{x^2}{2}\cdot\frac{1}{1+x^2} & 0 \le x < 1 \\ ? & \end{cases}$

$y(1) = \frac{1}{2}\cdot\frac{1}{1+1} = \frac{1}{4}$

2.3 cont.

$$y(1) = 1/4$$

$$(1+x^2)\,dy/dx + 2xy = -x$$

$$\frac{d}{dx}\{(1+x^2)y\} = -x$$

$$(1+x^2)y = -x^2/2 + C_2$$

$y(1) = 1/4$ $\qquad (1+1^2)\,1/4 = -1^2/2 + C_2$

$$C_2 = 2/4 + 1/2 = 1$$

$$y(x) = \begin{cases} x^2/2 \;\; 1/1+x^2 & 0 \le x < 1 \\ (-x^2/2+1)\;\; 1/1+x^2 & x \ge 1 \end{cases}$$