# MA3520 Struthers Sp2016 Exam 2:

Name Grading Key

Last day for me to look at these exams is Friday April 1st.

Show integrals etc. you enter in calculators.
Explain what you do. No imaginary numbers in answers.
desolve is not an acceptable technique. Read the instructions.
Use the back of a page to show additional work.

**1.** Solve $\frac{dP}{dt} = \frac{2}{t}P - t$ $\quad P(3)=1$

$dP/dt - 2/t\, P = -t$

$\mu(t) = e^{-2\ln(t)} = t^{-2}$ (+5)

1st order linear

(+5) $t^{-2}\, dP/dt - 2t^{-3}P = -t^{-1}$

$(P t^{-2})' = -t^{-1}$ (+5)

$P t^{-2} = -\int t^{-1}\, dt + C$

$\boxed{P t^{-2} = -\ln(t) + C}$ (+5)

$(1)\ 3^{-2} = -\ln(3) + C$

$\boxed{C = 1/9 + \ln(3)}$ (+5)

Grading Notes:

- need correct sol to get I.C.
- 5pts Integrating Factor
- 5pts knowing what to do with I.F.
- 5pts each side Integration and +C
- 5pts value of C

$C = 1.20972$ is ok

**2.** Solve $\left(y^2 e^{xy^2} + \cos(x)\right) dx = -\left(2xy e^{xy^2} + \frac{2}{4y^2+1}\right) dy$ $\quad y(\pi/2)=0$

| Q1 | Q2 | Q3 | Q4 | Total |
|---|---|---|---|---|
| | | | | |

$M\,dx + N\,dy = 0$

$M = y^2 e^{xy^2} + \cos(x)$

$N = 2xy e^{xy^2} + \frac{2}{(2y)^2+1}$

$\partial M/\partial y = \partial N/\partial x = 2y e^{xy^2} + y^2 2xy e^{xy^2}$ (+5)

Exact

$\partial S/\partial x = y^2 e^{xy^2} + \cos(x)$

$\Rightarrow S = e^{xy^2} + \sin(x) + g(y)$ (+5)

$\Rightarrow \partial S/\partial y = 2xy e^{xy^2} + 0 + g'(y) = 2xy e^{xy^2} + \frac{2}{(2y)^2+1}$

$g'(y) = \frac{2}{(2y)^2+1}$ (+5) $\Rightarrow g(y) = \text{ArcTan}[2y]$ (+5)

$$e^{xy^2} + \sin(x) + \text{ArcTan}[2y] = C$$

$$C = e^{(\pi/2)0^2} + \sin(\pi/2) + \text{ArcTan}[0] = 2$$ (+5)

Grading Notes

- correct sol^n to test 25/25
- correct S to test 20/25
- need correct I.C. for

**3.** Solve $y' + 2xy = xe^{-x^2}$

1st order linear

Score / 25

$\mu(x) = e^{x^2}$ (+5)

$e^{x^2}y' + 2xe^{x^2}y = xe^{-x^2}e^{x^2} = x$ (+5)

$(e^{x^2}y)' = x$ (+5)

$e^{x^2}y = \int x\,dx = x^2/2 + C$

$\boxed{e^{x^2}y = x^2/2 + C}$ (+10)

Grading Notes

**4.** Use Euler method to obtain a 4 decimal approximation of $y(2.3)$ for
$x\,y' = -2\,y + 1$ with $y(2.2) = 0.1$.
First use $h = 0.1$ and then $h = 0.05$. The solution of the IVP is
$y = 0.5 - 1.936\,x^{-2}$

| Score / 25 |
|---|
| |

| $x_n$ | $y_n$ | Actual Value | Abs Error | % Relative Error |
|---|---|---|---|---|
| 2.20 | | | | |
| 2.30 | +5 | | | |
| | | | | |
| | | | | |

| $x_n$ | $y_n$ | Actual Value | Abs Error | % Relative Error |
|---|---|---|---|---|
| 2.20 | | | | |
| 2.25 | (+5) | | | |
| 2.30 | (+5) | | | |
| | | | | |

| | $x_n$ | $y_n$ | Actual Value | Abs. Error | %Relative Error |
|---|---|---|---|---|---|
| 1 | 2.2 | 0.1 | 0.1 | 0. | 0. |
| 2 | 2.3 | 0.136364 | 0.134026 | 0.00233717 | 1.74381 |

| | $x_n$ | $y_n$ | Actual Value | Abs. Error | %Relative Error |
|---|---|---|---|---|---|
| 1 | 2.2 | 0.1 | 0.1 | 0. | 0. |
| 2 | 2.25 | 0.118182 | 0.11758 | 0.000601571 | 0.511626 |
| 3 | 2.3 | 0.135152 | 0.134026 | 0.00112505 | 0.839424 |

Grading notes
- 5 pts for each non-trivial # in $y_n$ col
- 5 pts for all values in Actual value column
- need everything else correct to get remaining points