Sp2016 Exam 3:

Name KEY

Show polynomials etc. you enter in calculators and explain what you did.

No imaginary numbers in final answers.

Read the instructions. I am frequently trying to save you time.

Use the back of a page to show more work if you need to.

4 questions - 25 points each.

1. Solve $y'' + 4y' + 4y = 48e^{2t}$ with $y[0] = 4,\ y'[0] = 8$

$m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2$ twice

$y_c = c_1 e^{-2t} + c_2 e^{-2t} t$

$y_p = Ae^{2t}$

$y_p' = 2Ae^{2t}$

$y_p'' = 4Ae^{2t}$

$(4A + 8A + 4A)e^{2t} = 48e^{2t}$

$16A = 48$ $\quad \boxed{A = 3}$

$y = c_1 e^{-2t} + c_2 te^{-2t} + 3e^{2t}$

$4 = c_1 + 3 \Rightarrow \boxed{c_1 = 1}$

$y' = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 te^{-2t} + 6e^{2t}$

$8 = -2c_1 + c_2 + 6 \Rightarrow \boxed{c_2 = 8 - 6 + 2c_1 = 4}$

Grading

5pts each $\boxed{c_1 e^{-2t}}$ $\boxed{c_2 te^{-2t}}$

5pts $y_p = Ae^{2t}$ + Attempt to sub

5pts $\boxed{A = 3}$

2pts $4 = c_1 + A$

3pts $\boxed{c_1 = 1 \quad c_2 = 4}$

* messed up y_c

2. Solve $y''+2y'+5y = 50t+17\sin(2t)$

Q1	Q2	Q3	Q4	Total

$m^2+2m+5=0$

$m=-1\pm 2I$

$y_c = c_1 e^{-t}\cos(2t) + c_2 e^{-t}\sin(2t)$

$y_{p_1} = At+B$ $\quad 0+2A+5(At+B)=50t+0$

$y_{p_1}' = A$ $\quad 5A=50$ $\quad \boxed{A=10}$

$y_{p_1}'' = 0$ $\quad 2A+5B=0$ $\quad 5B=-20$ $\quad \boxed{B=-4}$

$y_{p_2} = F\sin(2t)+G\cos(2t)$

$y_{p_2}' = 2F\cos(2t)-2G\sin(2t)$

$y_{p_2}'' = -4F\sin(2t)-4G\cos(2t)$

$-4F\sin(2t) - 4G\cos(2t)$

$-4G\sin(2t) + 4F\cos(2t)$

$+5F\sin(2t) + 5G\cos(2t)$

$17\sin(2t) + 0\cos(2t)$

$F-4G=17$ $\quad G+4F=0$

$G=-4F$

$F-4(-4F)=17 \Rightarrow \boxed{F=1}$ $\quad \boxed{G=-4}$

$$\boxed{y = y_c + 10t - 4 + \sin(2t) - 4\cos(2t)}$$

Grading: 5 pts each $c_1 e^{-t}\cos(2t)$ and $c_2 e^{-t}\sin(2t)$

5 pts y_{p_1} 2 pts form, 3 pts value

10 pts y_{p_2} 5 pts form, 5 pt values

3. Solve $y''' + 3y'' + 3y' + y = 6e^{-t}$

$y = y_c + y_p$

Score / 25

5pts each piece

$m^3 + 3m^2 + 3m + 1 = 0$

$(m+1)^3 = 0$

$m = -1$ three times

$y_c = c_1 e^{-t} + c_2 t e^{-t} + c_3 t^2 e^{-t}$

$y_p = A t^3 e^{-t}$ — 5pts Form

$y_p' = 3A t^2 e^{-t} - A t^3 e^{-t}$

$y_p'' = 6A t e^{-t} - 3A t^2 e^{-t} + A t^3 e^{-t} - 3A t^2 e^{-t}$

$y_p''' = 6A e^{-t} - 12A t e^{-t} + 3A t^2 e^{-t} - 6A t e^{-t} + 6A t^2 e^{-t} - A t^3 e^{-t}$

$y_p''' = (6A - 18A t + 9A t^2 - A t^3) e^{-t}$

$(6A - 18At + 9At^2 - At^3) e^{-t}$

$+3(\quad 6At - 6At^2 + At^3) e^{-t}$

$+3(\quad 3At^2 - At^3) e^{-t}$

$+(\quad At^3) e^{-t}$

$(6A + 0At + 0At^2 + 0At^3) e^{-t} = 6e^{-t}$

$6A = 6$ $\quad A = 1$

1 5pts value

4.

4.1. Show that $y_1 = t^{-2}$ is a solution of $t^2 y'' + 5ty' + 4y = 0$.

4.2. Solve $t^2 y'' + 5ty' + 4y = 50t^3$.
Make sure you write down equations you are solving and explain your process and steps.

Score / 25

$y_1 = t^{-2}$

$y_1' = -2t^{-3}$ (2 pts)

$y_1'' = 6t^{-4}$ (2 pts)

$t^2(6t^{-4}) + 5t(-2t^{-3}) + 4t^{-2} = (6-10+4)t^{-2} = 0$

y_1 is a soln (2 pts)

$y = ut^{-2}$ (5 pts)

$y' = u't^{-2} - 2t^{-3}u$ (2 pts)

$y'' = u''t^{-2} - 4t^{-3}u' + 6t^{-4}u$ (2 pts)

$t^2(u''t^{-2} - 4t^{-3}u' + 6t^{-4}u)$

$+5t(t^{-2}u' - 2t^{-3}u)$

$+4(t^{-2}u)$

$u'' + t^{-1}(-4+5)u' + t^{-2}(6-10+4)u = 50t^3$

$u'' + t^{-1}u' = 50t^3$ $\quad \mu(t) = e^{\int 1/t\,dt} = t$

(5 pts) $dw/dt + 1/t\, w = 50t^3$

$(tw)' = 50t^4$

$tw = 50/5\, t^5 + c_2$ $\quad \boxed{u' = 10t^4 + c_2/t}$ (3 pts)

$\boxed{u = 10/5\, t^5 + c_2 \ln(t) + c_1}$

(2 pts)