

Q1 / 12	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total

Struthers: MA3520 Final

Name _____

Show polynomials matrices etc. you enter in calculators, no points for dsolve, no imaginary answers.
Put extra work on the back of pages.

1. Solve

$$x' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \cdot x + \begin{pmatrix} 2+6t \\ 1 \end{pmatrix} \text{ with } x(0) = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

2. Solve

$$X' = \begin{pmatrix} 3 & 5 \\ -5 & 3 \end{pmatrix} X + \cos(2t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

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3. Solve the Ordinary Differential Equation

$$X' = \begin{pmatrix} -24 & -24 & 32 \\ -14 & -8 & 16 \\ -27 & -26 & 36 \end{pmatrix} X + e^{2t} \begin{pmatrix} 6 \\ 2 \\ 7 \end{pmatrix}. \text{ Mathematica gives eigenvalues as}$$

$\{-4, 4, 4\}$ and eigenvectors as $\{\{2, 1, 2\}, \{4, 2, 5\}, \{0, 0, 0\}\}$.

Fill in the matrix you row-reduce to satisfy $x[0] = \begin{pmatrix} 7 \\ 13 \\ -2 \end{pmatrix}$.

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4. Solve $xy' - 2x^2y = 6e^{x^2}$ with $y[1] == 2$

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5. Solve $(3y^2 + 2xy \cos(xy^2)) \frac{dy}{dx} = -(2x + y^2 \cos(xy^2))$ with $y(1) = 0$.

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6. Use Euler method to obtain a 4 decimal approximation of $y(1.2)$ for $y' = -2y + e^x$ with $y(1) = 1.2$.
First use $h = 0.2$ and then $h = 0.1$. Find an explicit solution for the initial value problem and fill in the tables.

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	x_n	y_n	Actual Value	Abs. Error	%Relative Error
1	1	1.2	0.3	0.	0.
2	1.2				

	x_n	y_n	Actual Value	Abs. Error	%Relative Error
1	1	1.2	0.3	0.	0.
2	1.1				
3	1.2				

7. Solve $y'' + 2y' + y = t + e^{-t}$

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- 8.** Show that $y_1 = t^{-1}$ solves $2t^2 y'' + 5ty' + y = 0$.
Solve $2t^2 y'' + 5ty' + y = \ln(t)$ with $y(1) = 1$ and $y'(1) = 2$ given that $y_c = c_1 y_1 + c_2 t^{-0.5}$ is the complementary solution. Show equations and explain process.

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