

## Sp2016 Practice Exam 2:

Name KEY

Show matrices etc. you enter in calculators and explain what you did.

No imaginary numbers in final answers.

Read the instructions. I am frequently trying to save you time.

Use the back of a page to show more work if you need to.

4 questions - 25 points each.

1. Solve  $\frac{dp}{dt} = p - p^2$   $P(0)=2$

$$\int \frac{dp}{p(1-p)} = \int dt = t + C_1$$

$$\frac{1}{p(1-p)} = \frac{A}{p} + \frac{B}{1-p} = \frac{1}{p} + \frac{1}{1-p}$$

$$\begin{cases} 1 = A(1-p) + Bp = A + p(B-A) \\ A=1 \quad B-A=0 \end{cases}$$

$$\ln|p| - \ln|1-p| = t + C_1$$

$$\ln|2| - \ln|1-2| = 0 + C_1$$

$$C_1 = \ln|2|$$

2. Solve  $\frac{dy}{dx} = x\sqrt{1-y^2}$

$y(1) = 1/2$

Q1	Q2	Q3	Q4	Total

$$\int \frac{dy}{\sqrt{1-y^2}} = \int x dx$$

$$\int \frac{1}{\sqrt{1-y^2}} dy = \frac{x^2}{2} + C$$

$$\boxed{\text{ArcSin}(y) = \frac{x^2}{2} + C}$$

$$\text{ArcSin}(1/2) = 1/2 + C$$

$$\boxed{C = -\left(\frac{1}{2} - \text{ArcSin}\left(\frac{1}{2}\right)\right)}$$

#39 in table

3. Solve  $\frac{dT}{dt} = k(T - T_m)$        $T(0) = T_0$

$$\frac{dT}{dt} - kT = -kT_m$$

$$u(t) = e^{-kt}$$

$$e^{-kt} T' - k e^{-kt} T = -k T_m e^{-kt}$$

$$(e^{-kt} T)' = -k T_m e^{-kt}$$

$$e^{-kt} T = T_m e^{-kt} + C$$

$$e^0 T_0 = T_m e^0 + C$$

$$C = T_0 - T_m$$

or sep

$$\int \frac{dT}{T - T_m} = \int k dt = kt + C_2$$

$$\ln |T - T_m| = kt + C_2$$

$$\ln |T_0 - T_m| = k \cdot 0 + C_2 = C_2$$

$$C_2 = \ln |T_0 - T_m|$$

Score / 25

4. Solve  $(x+1)y' + y = \ln(x)$        $y(1) = 10$ 

$$y' + \left(\frac{1}{x+1}\right)y = \frac{1}{x+1} \ln(x)$$

Score / 25

$$v(x) = e^{\ln(x+1)} = x+1$$

$$(x+1)y' + y = \ln(x)$$

$$\left((x+1)y\right)' = \ln(x)$$

$$(x+1)y = \int \ln(x) dx + C$$

$$(x+1)y = x \ln(x) - x + C$$

$$(1+1)10 = 1 \ln(1) - 1 + C$$

$$y(1) = 10$$

$$C = 21$$

5. Solve  $y' - \sin(x)y = 2 \sin(x)$   $y(\pi/2) = 1$

$$v(x) = e^{\int -\sin(x) dx} = e^{\cos(x)}$$

Score / 25

$$y' e^{\cos(x)} - \sin(x) e^{\cos(x)} y = 2 \sin(x) e^{\cos(x)}$$

$$\left( e^{\cos(x)} y \right)' = 2 \sin(x) e^{\cos(x)}$$

$$e^{\cos(x)} y = 2 \int \sin(x) e^{\cos(x)} dx + C$$

$$u = \cos(x)$$

$$du = -\sin(x) dx$$

$$e^{\cos(x)} y = -2 \int e^u du + C$$

$$e^{\cos(x)} y = -2 e^u + C$$

$$e^{\cos(x)} y = -2 e^{\cos(x)} + C$$

$$y(\pi/2) = 1 \quad e^{\cos(\pi/2)} y = -2 e^{\cos(\pi/2)} + C$$

$$c = e^{\cos(\pi/2)} (1 + 2) = 3$$

6. Solve

$$\left(\frac{1}{4+y^2} + \cos(x) - 2xy\right) \frac{dy}{dx} = y(y + \sin(x)) \quad y(0) = 1$$

Score / 25

$$(y^2 + y \sin(x)) dx - \left(\frac{1}{4+y^2} + \cos(x) - 2xy\right) dy = 0$$

$$M = y^2 + y \sin(x)$$

$$N = -\left(\frac{1}{4+y^2} + \cos(x) - 2xy\right)$$

$$\frac{\partial M}{\partial y} = 2y + \sin(x)$$

$$\frac{\partial N}{\partial x} = -\left(0 - \sin(x) - 2y\right) = 2y + \sin(x)$$

Exact



$$\frac{\partial s}{\partial x} = y^2 + y \sin(x)$$

$$\Rightarrow s = xy^2 - y \cos(x) + g(y) = N$$

$$\Rightarrow \frac{\partial s}{\partial y} = 2xy - \cos(x) + g'(y) = N$$

$$= 2xy - \cos(x) - \frac{1}{4+y^2}$$

$$g'(y) = -\frac{1}{4+y^2}$$

$$g(y) = -\frac{1}{2} \arctan\left(\frac{y}{2}\right)$$

$$xy^2 - y \cos(x) - \frac{1}{2} \arctan\left(\frac{y}{2}\right) = C$$

$$0(1^2) - 1 \cos(0) - \frac{1}{2} \arctan\left(\frac{1}{2}\right) = C$$

$$C = -\frac{1}{2} \arctan\left(\frac{1}{2}\right) - 1$$

7. Solve

$$(2y \sin(x) \cos(x) - y + 2y^2 e^{xy^2}) dx = (x - \sin(x)^2 - 4xy e^{xy^2}) dy$$

$$(2y \sin(x) \cos(x) - y + 2y^2 e^{xy^2}) dx$$

$$- (x - \sin(x)^2 - 4xy e^{xy^2}) dy = 0$$

Score / 25

$$\frac{\partial M}{\partial y} = 2 \sin(x) \cos(x) - 1 + 4y e^{xy^2} + 2y^2 e^{xy^2} \cdot 2xy$$

$$\frac{\partial N}{\partial x} = - (1 - 2 \sin(x) \cos(x) - 4y e^{xy^2} - 4xy e^{xy^2} \cdot y^2)$$

EXACT

$$\frac{\partial S}{\partial x} = 2y \sin(x) \cos(x) - y + 2y^2 e^{xy^2}$$

$$S = 2y \int \sin(x) \cos(x) dx - xy + 2e^{xy^2} + g(y)$$

$u = \sin(x)$   
 $du = \cos(x) dx$

$$S = y (\sin(x))^2 - xy + 2e^{xy^2} + g(y)$$

$$\frac{\partial S}{\partial y} = \sin(x)^2 - x + 2e^{xy^2} \cdot 2xy + g'(y)$$

$$= \sin(x)^2 - x + 4xy e^{xy^2}$$

$$g'(y) = 0 \Rightarrow g(y) \text{ is a constant}$$

$$y \sin(x)^2 - xy + 2e^{xy^2} = C$$

8. Use Euler method to obtain a 4 decimal approximation of  $y(1.2)$  for  $y' = 3y + 2e^x$  with  $y(1) = 0.3$ .  
First use  $h = 0.1$  and then  $h = 0.05$ . Find an explicit solution for the initial value problem and fill in the tables.

Score / 25

```
In[4] = ySol[x_] = Expand[ y[x] /. DSolve[ {y'[x] = 3 y[x] + 2 E^x, y[1] = 0.3}, y[x], x] [[1]] ]
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Out[4] = -1. e^x + 0.150271 e^{3 x}
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Out[77]//TableForm=

	$x_n$	$y_n$	Actual Value	Abs. Error	%Relative Error
1	1	0.3	0.3	0.	0.
2	1.1	0.933656	1.07009	0.136432	12.7496
3	1.2	1.81459	2.17955	0.364965	16.7449

Out[80]//TableForm=

	$x_n$	$y_n$	Actual Value	Abs. Error	%Relative Error
1	1	0.3	0.3	0.	0.
2	1.05	0.616828	0.649092	0.0322639	4.97062
3	1.1	0.995118	1.07009	0.0749708	7.00604
4	1.15	1.4448	1.57542	0.130614	8.29074
5	1.2	1.97734	2.17955	0.20221	9.27759

$$y' - 3y = 2e^x \quad w(x) = e^{-3x}$$

$$(e^{-3x} y)' = 2e^x e^{-3x} = 2e^{-2x}$$

$$e^{-3x} y = -e^{-2x} + C$$

$$e^{-3} (0.3) = -e^{-2} + C$$

$$C = e^{-3} (0.3) + e^{-2} = 0.150271$$