Exam 1   Name:

Directions: Answer each question to the best of your ability. The number of points the question is worth follows the question. Calculators are allowed, but partial credit can only be given if proper work is shown.

1. Given that $\vec{v} = i - 6j - 2k$ and $\vec{w} = 4i + 2j - 4k$ on the given set of coordinate axes, find the following: (5 pts each)

   (a) $3\vec{v} - 2\vec{w}$

   (b) $\vec{v} * \vec{w}$

   (c) $\vec{w} \times \vec{v}$

   (d) The magnitude of $\vec{w}$, or $||\vec{w}||$

   (e) The unit vector parallel to $\vec{w}$ pointing in the opposite direction.

   (f) The angle between $\vec{v}$ and $\vec{w}$.

2. Consider each of the following statements. Write T if the statement is always true, and F if the statement is not always true. $\vec{a}$, $\vec{b}$, and $\vec{c}$ are non-zero vectors. (4 pts each)

(a) $(\vec{a} * \vec{b}) * \vec{c} = \vec{a} * (\vec{b} * \vec{c})$

(b) $(\vec{a} \times \vec{b}) * \vec{c} = -(\vec{b} \times \vec{a}) * \vec{c}$

(c) If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are parallel, then $(\vec{a} \times \vec{b}) * \vec{c} = 0$

(d) If $\vec{a} \times \vec{b} = \vec{0}$, then $\vec{a} \perp \vec{b}$

(e) $(\vec{a} - \vec{b}) * (\vec{a} + \vec{b}) = ||\vec{a}||^2 - ||\vec{b}||^2$

3. Given the points $P=(1,2,4)$, $Q=(2,1,1)$, and $R=(-1,0,2)$, answer the following: (5 pts each)

(a) What is the vector $\vec{n} = \vec{PQ} \times \vec{PR}$?

(b) What is the defining characteristic of $\vec{n}$?

(c) What is the equation of the plane containing P, Q, and R?

4. For what value(s) of λ are the vectors $\vec{v} = \lambda i - 3j + 3k$ and $\vec{w} = \lambda i + \lambda j - 6k$ perpendicular? (5 pts)

5. The displacement (in meters) of an object is represented by the vector $\vec{d} = 3i - j - 2k$, while the constant force (in Newtons) acting on the object is represented by the vector $\vec{F} = 2i - 3j + k$. Find the work done on the object. (5 pts)

6. Find the projection of vector $\vec{v} = 2i - 3j + k$ onto the vector $\vec{w} = 8i - 9j - 12k$. (5 pts)

7. An airplane flies southwest with an airspeed of 400 mph. While in the air, the plane encounters a wind that blows due south at a speed of 50 mph. Assume a coordinate system such that the positive x and y are aligned with the East and North directions respectively. (5 pts each)

   (a) On the axes below, sketch the graphs of the vectors that represent the speed and direction of both the plane and the wind.
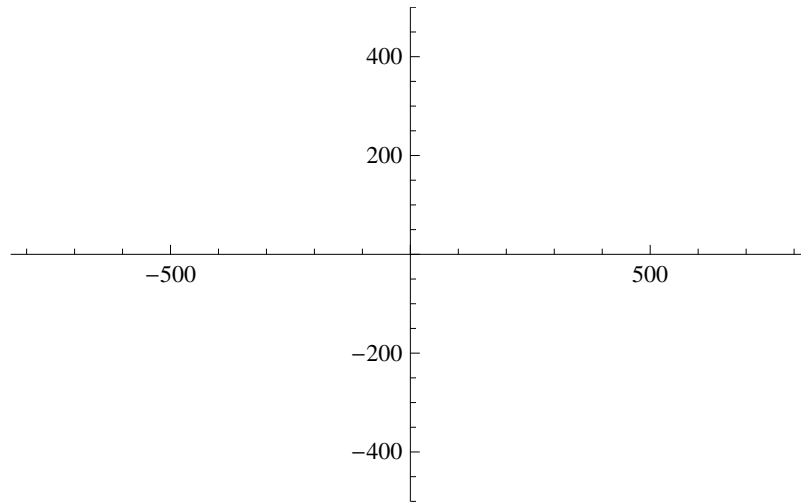
   (b) Express in component form the vector that represents the velocity of the airplane relative to the ground.

   (c) What is the speed of the airplane relative to the ground?

   (d) In which direction is the plane traveling relative to the ground? (express your answer in term of degrees relative to West)

4