Exam 3   Name: _____

Directions: Answer each question to the best of your ability. The point value of each question follows the question. Calculators are allowed, but you must show appropriate work to receive full credit.

Choose the best response for the following 10 questions. (4 pts each)

1. When taking a slice with the width dx, which of the following statements are true?

    (a) The slice is parallel to the x-axis

    (b) The slice is perpendicular to x-axis

    (c) The slice is perpendicular to the y-axis

    (d) The orientation of the slice cannot be determined

2. Which type of slice will you use to find the volume of revolution of $f(x) = x^2 - x + 1$ on [0,4] around the x-axis?

    (a) Circle or Disk

    (b) Washer

    (c) Cylindrical Shell

    (d) None of the above

3. Which is the correct integral to find the volume of revolution of the area between $f(x) = x^2$ and $g(x) = x$ on [0,1] revolved around the x-axis?

    (a) $\int_0^1 \pi(x^2)^2 \, dx$

    (b) $\int_0^1 \pi(x^4 - x^2) \, dx$

    (c) $\int_0^1 2\pi x(x - x^2) \, dx$

    (d) $\int_0^1 \pi(x^2 - x^4) \, dx$

4. Which of the following integrals will <u>NOT</u> find the volume of revolution of the area between $f(x) = x^2$ and $g(x) = x$ on [0,1] revolved around the y-axis?

    (a) $\int_0^1 2\pi x(x - x^2) \, dx$

    (b) $\int_0^1 \pi(y - y^2) \, dy$

    (c) $\int_0^1 \pi(x^2 - x^4) \, dx$

    (d) All three integrals will find the correct area

5. To find $\int x\,e^x$, which of the following methods will produce the correct answer

    (a) Substitution where O.F. $= e^u$ and I.F. $= u = x$

    (b) Integration by parts where $u = e^x$ and $v' = x$

    (c) Integration by parts where $u = x$ and $v' = e^x$

    (d) None of these previous methods will produce the correct answer

6. A square pyramid has a base area of 144 m^2 and a height of 24 m. If you slice the pyramid parallel to the base, which is the correct formula for the volume of a slice in terms of the height, h, from the apex of the pyramid?

   (a) $V = \frac{1}{4}h^2\,dh$

   (b) $V = \frac{1}{2}h^2\,dh$

   (c) $V = 2h^2\,dh$

   (d) $V = 4h^2\,dh$

7. We have learned two formulas to find the work done on an object, one using vectors and calculating the dot product and the other finding the definite integral of the force function with respect to position. What condition would compel us to use the second formula?

   (a) The displacement vector has a negative magnitude

   (b) The force is not constant

   (c) The force acts perpendicular to the displacement

   (d) The force acts parallel to the displacement

8. To find $\int \sin x \sqrt{\cos x}$, which of the following methods will produce the correct answer?

   (a) Integration by Parts where $u = \sin x$ and $v' = \sqrt{cosx}$

   (b) Substitution where O.F.$= \sqrt{u}$ and I.F.$= u = \cos x$

   (c) Integration by Parts where $v' = \sin x$ and $u = \sqrt{cosx}$

   (d) None of the previous three methods will provide the correct answer

9. Given the population density in terms of distance from a city's downtown is $\delta(r)$, which is the correct integral to find the total population living with 15 mi in all directions from downtown?

   (a) $\int_0^{15} 2\pi\delta(r)\,dx$

   (b) $\int_0^{15} \delta(r)\,dr$

   (c) $\int_0^{15} \pi(\delta(r))^2\,dx$

   (d) None of the previous three integrals will find the correct total population

10. A 100lb weight is suspended from the roof of a building by a 50ft chain which weighs 2lb/ft. Which integral will find the total work done to lift the weight and chain onto the roof?

    (a) $\int_0^{50} 100\,dx$

    (b) $\int_0^{50} 100 + 2x\,dx$

    (c) $\int_0^{50} 100 + 2(50 - x)\,dx$

    (d) The work cannot be determined as we need to know the height of the building first.

11. Find the area between $f(x) = x^2$ and $g(x) = 4x$ (Hint: find where the two functions intersect) (6 pts)

12. Find the volume of revolution of the area between $f(x) = x^2$ and $g(x) = 4x$ revolved around the x-axis (6 pts)

13. Find the volume of revolution of the area between $f(x) = x^2$ and $g(x) = 4x$ revolved around the y-axis (6 pts)

14. Find the volume of a square pyramid which has a base area of 144 m^2 and a height of 24 m using a Riemann sum converted to a definite integral. (6 pts)

15. Given the base of the pyramid is parallel to the yz-plane, and the apex of the pyramid is at the origin, find the coordinates of the center of mass of the pyramid. (6 pts)

16. A metal object fits the shape of the function $y = x^2$ revolved around the x-axis bounded by the interval $[0,10]$. If the function for the density of the object given the distance from the narrow end is $\delta(x) = \frac{100}{x^2}\ kg/m^3$, find:

   (a) The total mass of the object (6 pts)

   (b) The center of mass of the object (6 pts)

17. A 1000 lb block of ice is lifted 75 ft into the air. If the outside temperature causes the block to melt causing a loss of 3 lb for every foot lifted, what is the total work done to lift the block of ice? (6 pts)

18. A inverted cone-shaped tank is filled with water. The height of the cone is 20 feet and the radius at the top is 5 feet. How much work is necessary to pump all the water up out of the tank? (6 pts)

19. A dam has a trapezoidal shape with a top width of 500 feet, bottom width of 300 feet, and height of 100 feet. Assuming the dam is at capacity, what is the force of the water pressing against the dam? (6 pts)