Exam 4 Name: _____

Directions: Answer each question to the best of your ability. The point value of each question follows the question. Calculators are allowed, but you must show appropriate work to receive full credit.

Choose the best response for the following 9 questions. (5 pts each)

1. In the series, $a + ar + ar^2 + r^3 + ...$, for which values of r will the sum have a finite value?

   A. $\frac{2}{3}$ B. 5 C. $-\frac{4}{3}$ D. $-\frac{1}{5}$ E. 1

   (a) A, B, E
   (b) A
   (c) A, C
   (d) A, D

2. Which of the following series are geometric?

   A. $1 + \frac{5}{6} + \frac{2}{3} + \frac{1}{2} + ...$ B. $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + ...$ C. $\frac{2}{7} + \frac{3}{14} + \frac{4}{28} + \frac{5}{56}...$ D. $\frac{3}{4} + \frac{9}{20} + \frac{27}{100} + \frac{81}{500}...$

   (a) A, B
   (b) C, D
   (c) B, D
   (d) A, C

3. When the pump goes online, oil is retrieved from a well at a rate of 10,000 brl/day. However, each day of pumping reduces the volume retrieved by .2% per day. Which formula correctly finds the amount of oil retrieved at the end of 30 days?

   (a) $\frac{10{,}000(1-.002^{30})}{.002}$
   (b) $\frac{10{,}000(1-.998^{30})}{.002}$
   (c) $\frac{10{,}000(1-.998^{30})}{.998}$
   (d) $\frac{10{,}000(1-.002^{30})}{.998}$

4. What restriction exists for the Taylor polynomial to approximate a given function?

   (a) The function must be defined on the domain
   (b) The function must be positive-valued for all real numbers
   (c) The function must have a derivative that cycles as you take more derivatives
   (d) There is no restriction

5. What is the sum of the following geometric series? $2 - \frac{2}{5} + \frac{2}{25} - \frac{2}{125} + ...$

   (a) $\frac{5}{6}$

   (b) 1

   (c) $\frac{5}{3}$

   (d) $\frac{5}{2}$

6. Accounting for a possible 0^{th} term, which is the formula for the nth term of the following series? $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + ...$

   (a) $\frac{(-1)^{(n-1)}x^{2n}}{(n-1)!}$

   (b) $\frac{(-1)^{n}x^{2n}}{n!}$

   (c) $\frac{(-1)^{(n-1)}x^{2(n-1)}}{(n-1)!}$

   (d) $\frac{(-1)^{(n-1)}x^{2(n-1)}}{n!}$

7. Which is the correct Taylor series for e^x about $x = 0$

   (a) $1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \frac{x^4}{5!} + ...$

   (b) $1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...$

   (c) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$

   (d) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$

8. Which is the correct Taylor series for $\sin x$ about $x = 0$

   (a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$

   (b) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$

   (c) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...$

   (d) $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + ...$

9. Which is the defining characteristic of a Taylor polynomial of degree n that approximates $f(x)$ about $x = a$?

   (a) The coefficient for each term of the polynomial is nonzero.

   (b) The polynomial is an even function.

   (c) The polynomial has n real roots.

   (d) The evaluation of the 1^{st} to n^{th} derivatives at $x = a$ of the polynomial and $f(x)$ are the same.

10. State the formula for the degree 3 Taylor polynomial of $f(x)$ about $x = 0$ (5 pts)

11. Find the first 4 nonzero terms of the Taylor series about $x = 0$ for the following functions (5 pts each)

    (a) $\cos x$

    (b) $\frac{1}{(1+x)^3}$

12. Use your answer from 11a to estimate $\cos 1$ to 6 decimal places. (5 pts)

13. Use the Taylor polynomial of degree 4 about $x = 0$ for e^x to estimate e^1 to 6 decimal places. (5 pts)

14. Calculate the error on the degree 4 Taylor polynomial estimate of e^1 to 6 decimal places. (5 pts)

15. Find the necessary degree for a Taylor polynomial about $x = 0$ to calculate e^1 within .000001. (5 pts)

16. Using your answer to 15, estimate e^1 to 8 decimal places. (5 pts)

For the last part of the test, choose between pages 5 and 6. I will grade only one. If you do both, indicate which you want me to grade. For each of the applicable problems, use the first 4 non-zero terms for the series.

17. Express $f(x) = e^{x^3}$ as a Taylor series. (3 pts)

18. Verify using Taylor series that $f'(x) = 3x^2e^{x^3}$ (7 pts)

19. Give the formula for the nth term for the Taylor series for $3x^2e^{x^3}\,dx$. (5 pts)

20. Find the Taylor series for $\frac{1}{\sqrt{1+y}}$. (5 pts)

21. Use your answer to #20 to find the Taylor series for $\frac{1}{\sqrt{1-x^2}}$. (5 pts)

22. Use your answer to #21 to find the Taylor series for $\arcsin x$. (5 pts)