Now that we know how to find Taylor series, we can apply the principles of differentiation, integration, and substitution to find other Taylor series of functions that we would not be able to find

Goal

1. Apply the principles of differentiation, integration, and substitution to Taylor series to find other Taylor series.
2. Apply these principles to find integrals such as $\int e^{-x^2}\,dx$
3. Apply these principles to find values of interesting numbers such as $e^{\pi i}$

Since Taylor series are polynomials of infinite degree with a clearly discernible pattern for each term, we can apply the principle of substitution, differentiation, and integration to find other Taylor series. The most obvious example is the sine and the cosine. Just as the derivative of the sine is the cosine, the derivative of the Taylor series for the sine is the Taylor series of the cosine. Using these three principles, find the Taylor series for the following functions:

1. $f(x) = \sin x^2$
2. $f(x) = e^{-x^2}$ then find $\int e^{-x^2}\,dx$
3. $f(x) = \frac{1}{1+x^2}$ then find $f(x) = \arctan x$
4. $f(x) = e^{\cos x}$
5. $f(x) = e^{ix}$ then find $e^{\pi i}$