Class,

Here is the correct solution to example 5

What degree Taylor polynomial centered at $x = 0$ is necessary to guarantee $\ln 2$ to the nearest .000001?

The problem in class was that I was miscounting my derivatives

$f(x) = \ln 1 + x$

$f'(x) = (1 + x)^{-1}$

$f''(x) = -(1 + x)^{-2}$

. . .

$f^{(n+1)}(x) = n!(1 + x)^{-(n+1)}$

Note: we ignore the alternating negative because we are interested in absolute value.

The next question is which of these are bigger when evaluated over the interval. We said in class that as x got closer to 1, the denominators got bigger making the value smaller. This is true even though there is the factorial in the numerator. We didn't cover the math that goes behind this, but it is laid out in the end of 10-4 if you want to read it.

Therefore $f^{(n+1)}(x)$ is biggest when $x = 0$, therefore $M = n!$. This being so, recalling that we are analyzing $\ln 2$ so $x = 1$, we have the following equation:

$$\frac{M * x^{n+1}}{(n+1)!} = \frac{n! * 1^{n+1}}{(n+1)!} = \frac{1}{n+1} \leq \frac{1}{1,000,000}$$

This means $n \geq 999,999$. Remember, this does not mean that it is required to have 999,999 terms to be one millionth close. It says that you are guaranteed to be one millionth close with 999,999 terms in your polynomial