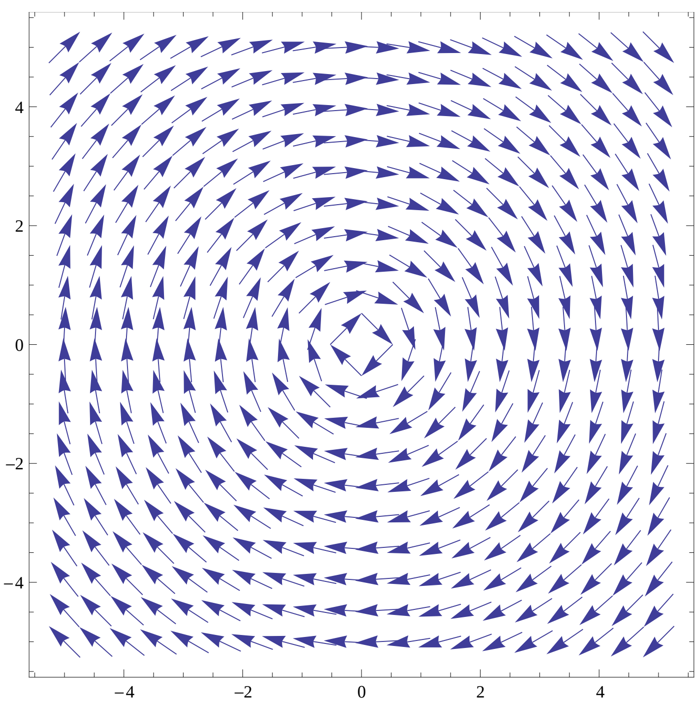
Today, we analyze the graph of the family of solutions. We call this graphing the slope field

Goal

1. Define a slope field
2. Generate a slope field
3. Graph possible solutions using a slope field.

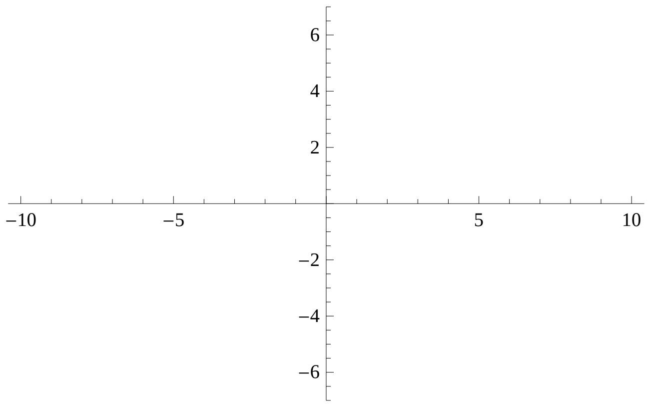
What is a slope field? A slope field is a graph on a coordinate plane of the values of y' given an equation for y'. Given the differential equation $y' = -\frac{x}{y}$, the slope field generated is the following graph:



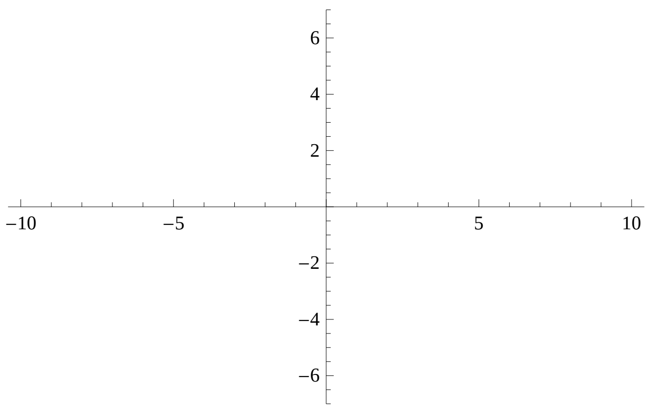
How is this generated? By taking various points, we calculate the slope using the differential equation, and draw a line accordingly.

For the following differential equations, graph the corresponding slope fields.

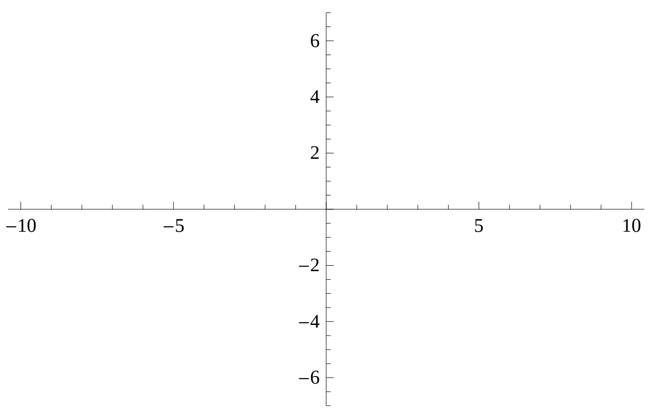
1. $y' = y$



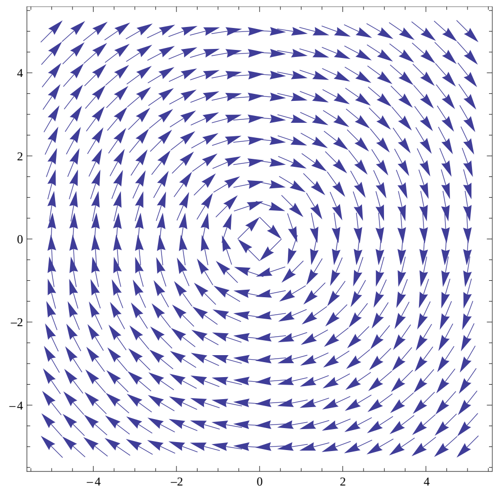
2. $y' = x$



3. $y' = 2 - y$



What good are slope fields? They help us in two respects. One, we can see a graph of what potential solutions look like; two, if we have a starting point, we can trace the solution and possibly determine the equation for a particular solution. Take for example our original slope field.



It seems obvious that the solution to this equation is a circle, which when we differentiate $x^2 + y^2 = r^2$, we find $y' = -\frac{x}{y}$.

With the following initial conditions, we can determine the actual equation:

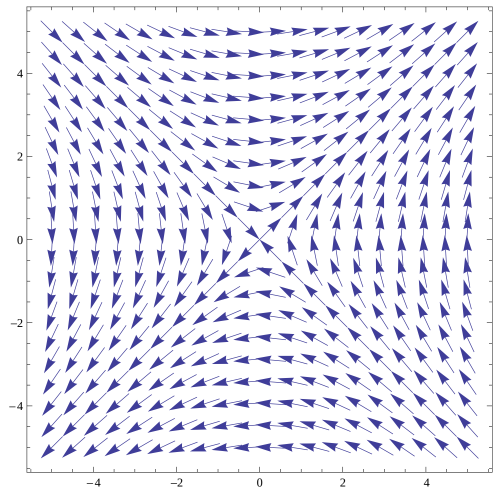
1. $y(0) = 4$; we get the graph of a circle with radius 4; hence the equation $x^2 + y^2 = 16$

2. $y(2) = 0$; we get the graph of a circle with radius 2; hence the equation $x^2 + y^2 = 4$

3. $y(0) = -3$; we get the graph of a circle with radius 3; hence the equation $x^2 + y^2 = 9$

On our three previous examples sketch the solution for the first when $y(0) = 1$, the second when $y(0) = 0$, and the third when $y(0) = -1$

What about the following differential equation $y' = \frac{x}{y}$ and its accompanying slope field:



If you think this looks like a hyperbola, you would be correct, as the solution of this equation is either $x^2 - y^2 = r^2$ or $y^2 - x^2 = r^2$.

Trace the solution which pass through each of the following points:

1. (1,0)

2. (0,1)

3. (-3,1)

4. (2,-1)

5. (1,1)