Now that we have found a slope field, how can we use it to approximate the graph of my function, y. We call this method Euler's method.

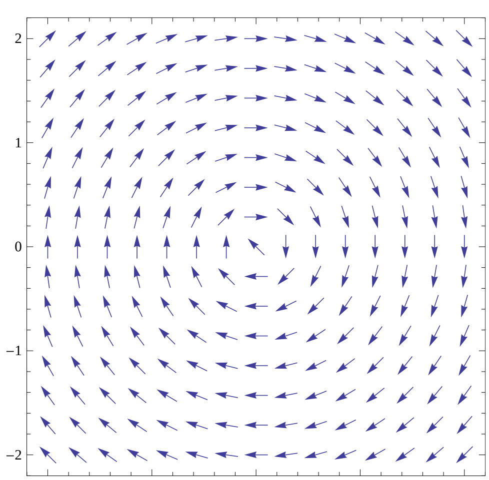
Goal

1. Define Euler's method
2. Use Euler's method to approximate the graph of y

When we have a slope field and an initial value, how can we approximate the graph of y? We use a method called Euler's method by which we use the slope at the point over a small interval to plot the next point on the graph. At the next point on the graph, we calculate the slope and plot the third point of the graph using the same width interval. We can continue this process until we have approximated our desired value of y for a given x value.

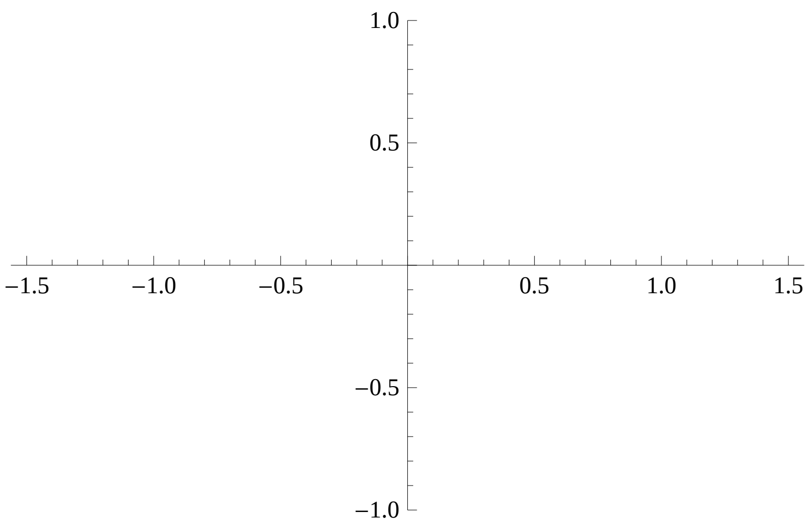
Obviously, there will be some error in this approximation, but how can we reduce that error? As we might expect, we reduce the error when we reduce the width of the interval or increase the number of intervals from the starting point to the desired point.

Example: Take the following slope field, $y' = -\frac{x}{y}$:

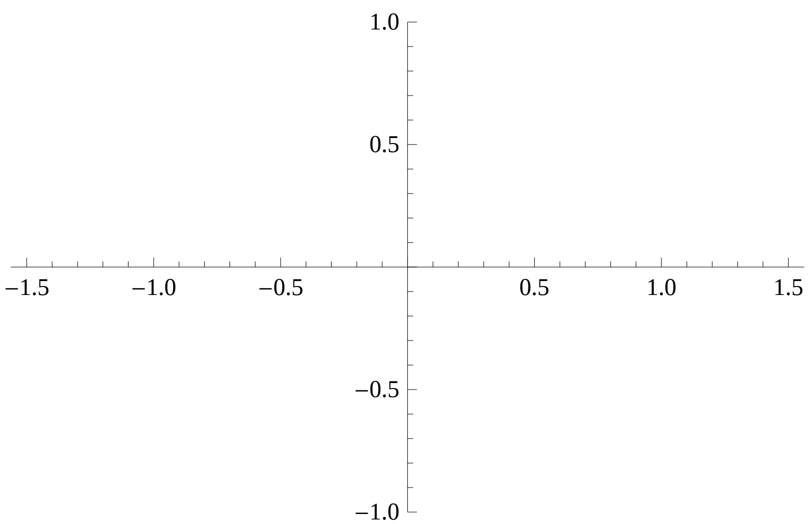


We know that this graph is a circle, but let's use the initial point of (0,1) to approximate the graph of y.

$y' = -\frac{x}{y}$, interval width .5

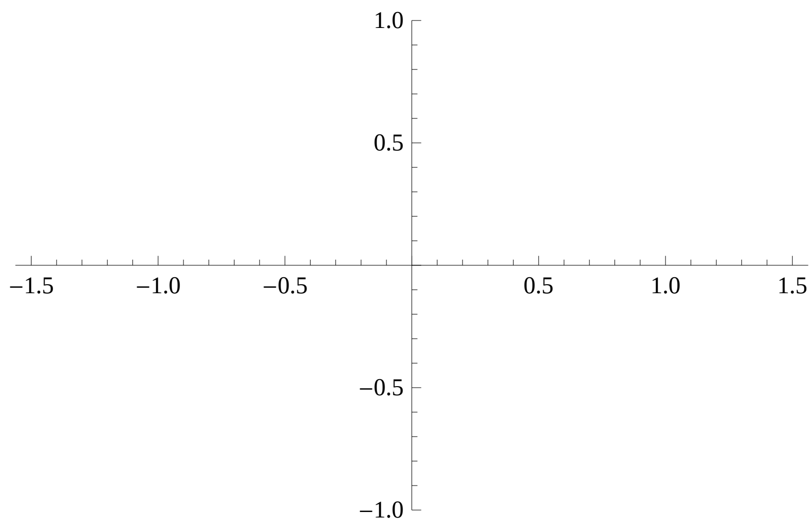


If we try again with an interval width of .2, we will get a much better result.

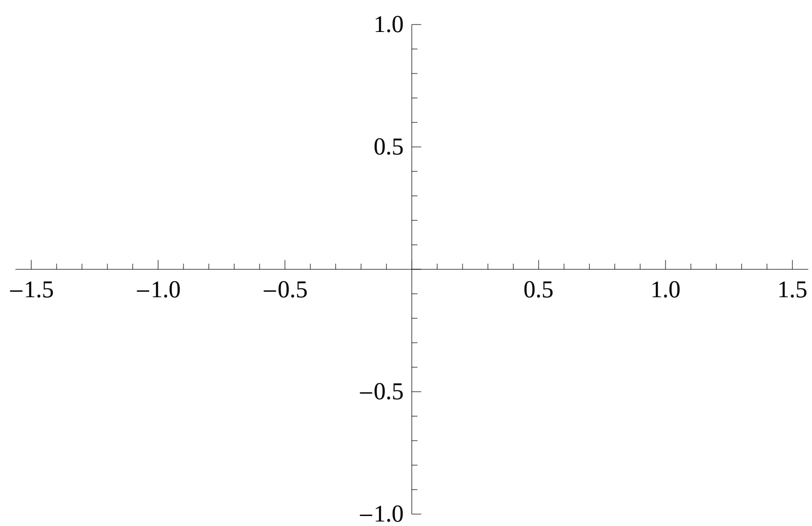


Let's try it again with a new differential equation.

$y' = -y$, interval width .5, initial point $(0, \frac{1}{2})$

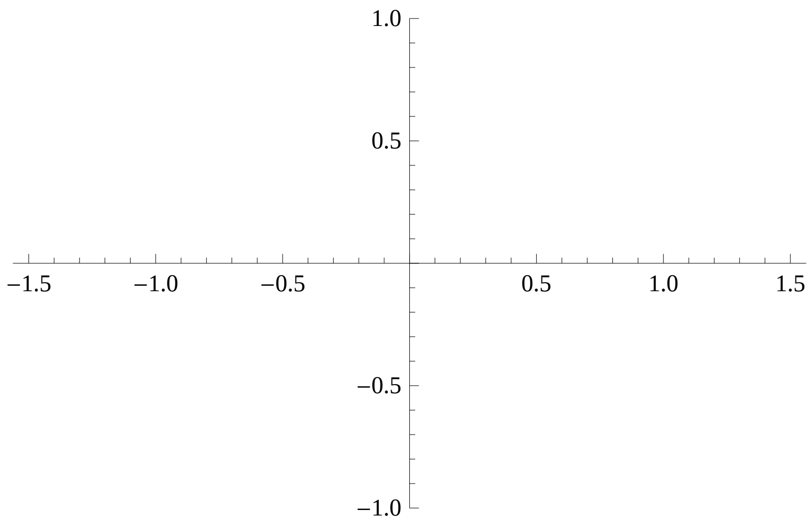


If we try again with an interval width of .2, we will get a much better result.



Let's try it again with a new differential equation.

$y' = xy$, interval width .5, initial point $(0, -\frac{1}{10})$



If we try again with an interval width of .2, we will get a much better result.