Now that we have approximated y from a differential equation, today we will learn how to find an equation for y given a differential equation. This is accomplished by separating the variables and integrating both sides of the equation

Goal

1. Solve differential equations for y
2. Find general solutions to common differential equations

Recall that a differential equation is an equation for y' in terms of x and y with the assumption that y is an implied function of x. Therefore, if we can combine the y and the y' parts of the equation in the proper method, we can use the fact that y is a function of x and y' is its implicit derivative to integrate both sides with respect to x. This only works if we have the y or function of y multiplied to y'. If this is the case, we apply the rule of substitution to integrate the y side, and we integrate the x side in the elementary way.

For example, we know the solution to $y' = y$ is $y = Ce^x$. We see this by starting with the differential equation, dividing by y, integrating both sides, and solving for y.

Find a general solution to the following differential equations:

1. $y' = ky$, where k is a nonzero constant.

2. $y' = xy$

3. $y' = -\frac{x}{y}$

4. $y' = \frac{x}{y}$

5. $y' = k - y$, where k is a non-zero constant

6. $y' = -k(y - a)$, where k and a are non-zero constants

The key to solve differential problems this way is to be able to separate the variables. As we have seen, this separation must take place through a process of multiplication or division. If the variables cannot be separated, the problem cannot be solved analytically.

Determine whether the variables can be separated in the following equations. If so, solve.

(a) $y' = x + y$

(b) $y' = x \tan y$

(c) $y' = x^y$

(d) $y' = xy^2$