Goal

1. Define integration by parts
2. Relate integration by parts to the product rule of differentiation
3. Solve problems using integration by parts

What is integration by parts? Integration by parts is defined by reverse engineering the product rule of differentiation:

$$\frac{d}{dx}uv = uv' + u'v$$

where u and v are functions of x

If we take the integral of this equation, we get:

$$uv = \int uv'\,dx + \int u'v\,dx$$

rewriting:

$$\int uv'\,dx = uv - \int u'v\,dx$$

This is the formula for integration by parts. This formula is used when we have an integral of the form:

$$\int f(x)g(x)\,dx$$

What is now required is to substitute $f(x)$ and $g(x)$ for u and v. The question now exists what principles guide the assignment to u and v. In general, use the following guidelines:

1. Choose as u something that differentiates to 0 or something simpler than u
2. Choose as v something that integrates indefinitely and stays the same complexity (if possible)

As a simple example, find the following integral:

$$\int x \sin x \, dx$$

Letting $u = x$ and $v' = \sin x$, we can fill plug in to the formula:

$$\int x \sin x \, dx = -x \cos x - \int 1 * -\cos x \, dx = -x \cos x + \sin x$$

The usefulness of this formula is that it can be applied multiple times to find a solution.

Problems to work on: Examples 4, 5, 7; Exercises 18, 25, 43