We can use the definite integral to find areas and volumes by applying a Riemann sum.

Goal

1. Apply a Riemann sum to estimate an area

2. Apply a definite integral to the Riemann sum to find exact area

Recall that we estimated area under the curve by drawing rectangles that approximated the area under the curve, and that we increased the accuracy of our estimate by taking the limit of the width of interval as it approaches 0. We can use the same approach to find the areas and volumes of a wide variety of shapes and figures. Admittedly, we know the formulas for many shapes from geometry, but our shape will not always be so nice

Steps to find Area

1. Determine orientation of the slice (or rectangle)–horizontal or vertical

2. If possible and helpful, write an equation that matches the shape analyzed and/or place on coordinate axes

3. Find area (or volume) of one slice.

4. Convert formula to variable that matches the slice

5. Write definite integral

6. Find definite integral

1. A right triangle has a height of 10 and a hypotenuse of length 20. Find the area taking horizontal and then vertical slices.

2. Find the area between the functions $9 - x^2$ and $(x - 3)^3$ from $x = 0$ to 3.

3. Find the area of a circle with radius 5 using vertical slices.

4. Given that the height of a cone is three times the radius, find the volume of a cone 15 cm tall taking horizontal slices.

5. Given that the side of the base of a square pyramid is the same as the height, find the volume of a square pyramid whose base is 100 m on a side.

6. The cross-section of a $100m$ tunnel is a semi-circle with radius $50m$. Find the volume of the tunnel using vertical and horizontal slices.