Again applying the principles of Riemann sum, we can find volumes of various kinds of looking shapes when revolving areas around the x- or y-axis.

Goal

1. Identify three shapes we can use to approximate the volume of a slice of volumes of revolution and what circumstances we use each one.
2. Apply a definite integral to the formulas of these shapes to find the exact volume.
3. Find the length of the arc of an interval of a function.

When we take an area and revolve it around either axis, we have methods of finding this volume of revolution. If the revolution is around the x-axis (or any horizontal line), we can use the shape of either a circle or washer to model one slice of the volume. If the revolution is around the y-axis (or other vertical line), we can use the shape of a cylindrical shell.

Steps to find volumes of revolution

1. Identify type of slice being used
2. Identify formula for the slice
3. Set up and find definite integral using appropriate limits

3 shapes of the slices

1. Circles (or cylinders)–Area is in contact with the horizontal axis of revolution
   (a) Volume of revolution of $f(x) = x^2$ on [0,4] around x-axis
   (b) Volume of revolution of $f(x) = x^3 + 2$ on [0,3] around x-axis
   (c) Volume of revolution of $f(x) = 2x + 3$ on [-2,5] around $y = 3$
2. Washers–Area is not in contact with horizontal axis of revolution
   (a) Volume of revolution of area between $f(x) = x^2$ and $g(x) = 8\sqrt{x}$ on [0,4] around x-axis
   (b) Volume of revolution of area between $f(x) = x^2$ and $g(x) = x^3$ on [0,1] around x-axis
   (c) Volume of revolution of area between $f(x) = x^2$ and $g(x) = x^3$ on [0,1] around line $y = 4$
3. Cylindrical Shells–Vertical axis of revolution
   (a) Volume of revolution of area between $f(x) = x^2 + 4$ and $g(x) = x^3$ on [0,2] around y-axis
   (b) Volume of revolution of area between $f(x) = x^2$ and $g(x) = x^3$ on [0,1] around y-axis
   (c) Volume of revolution of area between $f(x) = x^2$ and $g(x) = 8\sqrt{x}$ on [0,4] around $x = 4$

Arc length

Using the Pythagorean theorem, we can approximate the length of the curve with a Riemann sum. Therefore, we can find the exact value of this sum, and therefore the length of the arc over $[a, b]$ with the following formula:

$$\texttt{Arc Length} = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$