We can apply these integration techniques to the principles of physics regarding force, work, and pressure.

Goal

1. Identify an integral formula to find the work done given a function of force with respect to position
2. Identify a formula for the force exerted on an object given the pressure (whether constant or not)
3. Apply these formulas to solve problems

We know from our previous study that the formula for work is $W = F * d$ where in the event of the force being parallel to the displacement, we have straight multiplication, and when it is not, we take the dot product. This makes one large assumption: that the force is constant throughout its application. What if the force changes with respect to the position? What if, as the object moves, the force changes in intensity? How can we find the entire amount of work done throughout the total displacement of an object? As you might suspect, we need to take an integral (we are in the chapter of application of integrals), and the integral is as follows:

$$W = \int_a^b F(x)\,dx$$

where $F(x)$ is the equation for the force with respect to the position as represented by x.

Does this make sense based upon what we know? Absolutely. Imagine $F(x)$ under the conditions we already know. Given that force is a constant, $F(x) = K$, and $\int_a^b K\,dx = K(b-a)$ where K is the constant force and $b-a$ is the displacement. Hence $W = F * d$.

Remember when dealing with gravity, there is a difference between mass and weight. Mass is independent of the the force of gravity, while weight considers the force of gravity. So if a problem give us mass, we need to add the force of gravity to the equation; if it gives us weight, we do not.

Problems to work on: Exercises 9, 10, 11

Another set of application problem deal with pressure and force. Given a container with a base area of 1 square unit, the formula for pressure is:

$$P = \delta g h$$

where δ is the mass density of the substance, g is the acceleration due to gravity, and h is the height (or depth) of the container (the top is considered the 0 point).

Obviously if the density is not constant, we must include an integral relating the height and density to find the pressure $P = g \int_0^h \delta(h)\, dh$.

Remember that we are calculating this for a container with base area equal to one. What if the area is not one? then we get the total force with the formula.

$$F = P * \texttt{area}$$

Again, this assumes the pressure is constant throughout the area. If not, then we use an take and integral of the pressure with respect the correct dimension of the area. Therefore, given the area is in terms of x and y where x dimension has the variable pressure and $y = f(x)$ is the dimension with constant pressure, then the integral necessary is:

$$F = \int_a^b f(x) P(x)\, dx$$

where a and b are the positions of the start and end of the area in association with the coordinate axis.

Problems to work on: Exercises 23, 24