We can show from algebra that $.\bar{9} = 1$. But what is $.\bar{9}$? It is a geometric series, and its value is determined by the limit as the number of terms approaches infinity.

Goal

1. Define geometric series and partial sums
2. Identify the means and conditions to find the finite value of a geometric series.

What is a geometric series? It is the sum of an infinite set of numbers such that the ratio of any two consecutive numbers is constant. Using summation notation, where a is the first term, and n approaches ∞:

$$\Sigma_{i=1}^{n} ar^{i-1}$$

We define a partial sum as the sum of a consecutive finite set of the infinite set.

How do we find the value of the partial sum? Letting x equal the partial sum and performing some algebraic manipulation, we find the value:

$$x = \frac{a(1-r^n)}{1-r}$$

What if we want to find the sum of the entire series? We need to find the limit of the partial sum as $n \to \infty$:

$$\lim_{n\to\infty} \Sigma_{i=1}^{n} ar^{i-1}$$

It is quite obvious that if $|r| \geq 1$, there is no limit. However, if $0 < r < 1$, then the limit exists and the value of the sum is:

$$\Sigma_{i=1}^{n} ar^{i-1} = \frac{a}{1-r}$$

You would think that if $r < 0$, we may have to modify the formula, but it doesn't matter. As long as $|r| < 1$, the formula works.

Utilizing this method, we can further establish the fact that $.\bar{9} = 1$, as $a = \frac{9}{10}$ and $r = \frac{1}{10}$

Find the value of the geometric series given the following conditions:

1. $a = 1$, $r = \frac{1}{2}$
2. $a = \frac{2}{5}$, $r = \frac{1}{4}$
3. $a = 15$, $r = \frac{1}{3}$
4. $a = -16$, $r = \frac{2}{7}$
5. $a = 1$, $r = -\frac{3}{5}$