

■ **Basic Integrals**

Solve: (right or wrong)

a)  $\int x^{-2} dx$                       b)  $\int \cos[2x] dx$                       a)  $\int \frac{1}{x} dx$

■ **Substitution**

■ Given the integrals below, put an S by the integral if substitution is appropriate, and N if it is not. If the substitution is appropriate identify the appropriate substitution, but do not solve i.e.  $u = ?$

a) a)  $\int \frac{1}{x \log[x]} dx$                       b)  $\int x^3 \cos[x^2] dx$                       c) a)  $\int \frac{e^{\cos[x]}}{\sin[x]} dx$                       c)  $\int \sin[x] e^{\cos[x]} dx$

■ **Solve completely (right or wrong)**

$$\int x^2 \log[x^3] dx$$

■ **Limits of Integration, Solve completely: (right or wrong)**

$$\int_0^1 x \cos[x^2] dx$$

■ **Thought/Concept questions**

Integration by Substitution is related to which rule for differentiation?

- a) Sum Rule                      b) Product rule                      c) Chain Rule                      d) Quotient Rule

Given the functions  $g(x)$  and  $f(x)$ , then:  $\frac{d}{dx} f(g(x)) = f'(g(x)) * g'(x)$  which means that:  $f(g(x)) = ??$

■ **Integration by Parts**

Integration by parts is related to which rule for differentiation?

- a) Sum Rule                      b) Product rule                      c) Chain Rule                      d) Quotient Rule

Given the functions  $g(x)$  and  $f(x)$ , then:

$$\frac{d}{dx} u(x)v(x) = u'(x)v(x) + u(x)v'(x)$$

$$u(x)v(x) = \int u'(x)v(x) dx + \int u(x)v'(x) dx$$

and thus:  $\int u(x)v'(x) dx = ???$

■ Given the integrals below, put a BP by the integral if By Parts is appropriate, and N if it is not. If by parts is appropriate identify  $u$ , and  $dv$ , but do not solve ?

a) a)  $\int x \cos[x] dx$                       b)  $\int x^5 \cos[x^6] dx$                       c) a)  $\int x^2 e^x dx$                       c)  $\int \log[x] dx$

■ **Solve completely (right or wrong)**

$$\int x \cos[x] dx =$$

■ **Limits of Integration, Solve completely given that: (right or wrong)**

$$\int_0^\pi x \cos[x] dx =$$

■ **Partial Fractions**

Integration by partial fractions is an algebraic trick that allows us to simplify the integrand of some integrals?

- Given the integrals below, put a PF by the integral if Partial Fractions is appropriate, and N if it is not. If Partial Fractions express the partial fraction decomposition in terms of the unknown coefficients (do not solve for the coefficients) ?

a)  $\int \frac{x^3 - 2x^2 - 16}{x} dx$       b)  $\int \frac{2x+3}{x^3 - 2x^2 + 1x} dx$       c) a)  $\int \frac{x^2}{x^3 - 1} dx$       c)  $\int \frac{2}{(x-2)(x-1)^2} dx$

- Solve for the coefficients if: (right or wrong)

$$\frac{4x + 1}{(x - 1)(x + 3)} = \frac{A}{(x - 1)} + \frac{B}{(x + 2)}$$

- Solve completely (right or wrong)

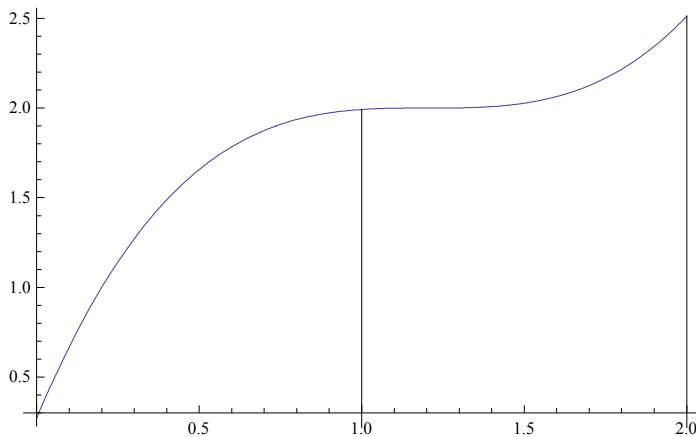
$$\int \frac{1}{(x - 2)(x - 3)} dx =$$

**Numerical Techniques**

Which of the following are reasons for using numerical integration techniques (select all that apply)?

- a) Integral may not have an elementary anti-derivative
- b) Numerical techniques are more accurate than analytic techniques
- c) Numerical techniques may be faster than analytic techniques on computers

- Given the picture, answer the following questions



Give the height of the first box using the following approximations.

- a) LHS
- b) RHS
- c) MID
- d) TRAP

What is the width of each box?

Draw and label the graphical representation of each technique (LHS, RHS, MID, TRAP) in the diagram.

In the first box, state whether the following approximations are over estimates, under estimates, or exact values.

- a) LHS
- b) RHS
- c) MID
- d) TRAP

■ Given the data below, answer the following questions:

distance	0.	0.5	1.	1.5	2.
height	0.272	1.657	1.992	2.027	2.512

- a) Approximate the value of the integral using the ?? technique  
 b) Why can't we use the midpoint approximation for this problem?

■ Error

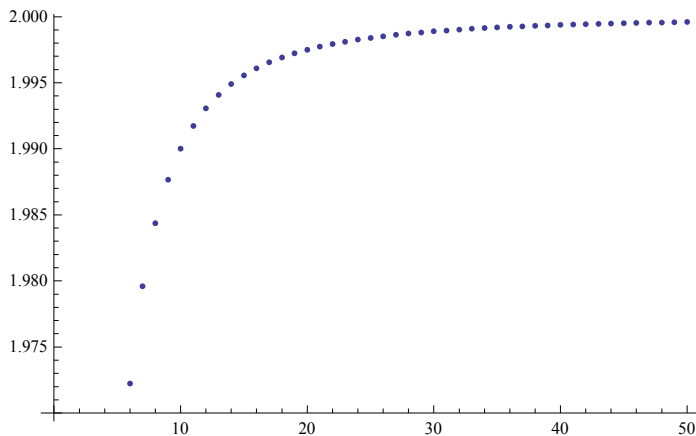
The error in the LHS estimate is proportion to  $\frac{1}{n}$ , what are the errors proportional to for the following techniques:

- a) RHS  
 b) Mid  
 c) Simp

If the error in the Trap method is proportion to  $\frac{1}{n^2}$ , and we double the number of step, then the error is reduced by what factor. i.e. new error =  $\frac{\text{old error}}{?}$

Given the graph below that shows the trap approximation and different step numbers,

- a) What do think the exact value of the integral is?  
 b) If i wanted an approximation that was good to two decimal places, about how many steps should i be taking?

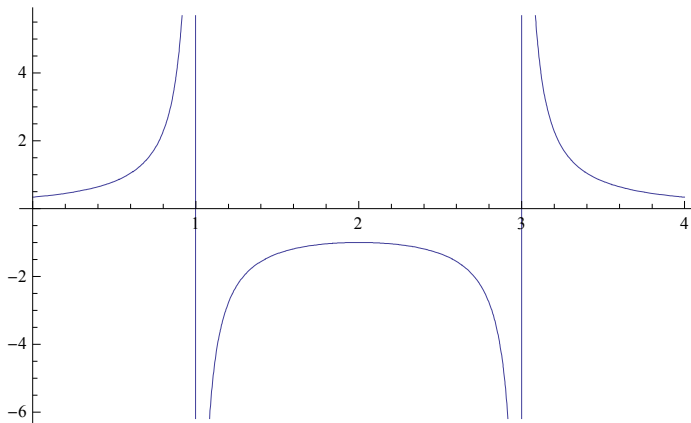


■ Improper Integrals

Which of the following are reasons why an integral would be improper? **this could be T or F, or list the two reasons**

- a) Integrand becomes unbounded  
 b) Integral may not have an elementary anti-derivative  
 c) Limits of integration go to infinity  
 d) Numerical techniques don't converge

- Given the picture of the function  $g[x]$  shown below, answer the following questions



Label each of the following integrals as proper or improper.

- $\int_0^{0.5} g[x] dx$
- $\int_{0.5}^{1.5} g[x] dx$
- $\int_{1.5}^{2.5} g[x] dx$
- $\int_{2.5}^{3.5} g[x] dx$
- $\int_{3.5}^{\infty} g[x] dx$

Label each of the following integrals as proper or improper.

- $\int_0^{10} \frac{1}{x-8} dx$
- $\int_2^3 \frac{x^2-3}{x-1} dx$
- $\int_1^5 e^{x-5} dx$
- $\int_1^{\infty} e^{x-5} dx$

Given that the  $\int x^{-2} dx = \frac{-1}{x}$ , re-write the following improper integral  $\int_0^1 x^{-2} dx = \frac{-1}{x}$  as the limit of a proper integral.

- Evaluate the limit, and give the solution to the integral