Note in Mathematica $Log[x] = ln(x)$

- **Basic Integrals**

Solve: **(right or wrong)**

a) $\int x^{-2}\,dx = -\frac{1}{x} + C$ b) $\int \text{Cos}[2x]\,dx =$ ~~sin~~ $\frac{1}{2}\sin(2x) + C$ a) $\int \frac{1}{x}\,dx = \ln(|x|) + C$

- **Substitution**

- **Given the integrals below, put an S by the integral if substitution is appropriate, and N if it is not. If the substitution is appropriate identify the appropriate substitution, but do not solve i.e. u = ?**

a) a) $\int \frac{1}{x\,\text{Log}[x]}\,dx$ b) $\int x^3\,\text{Cos}[x^2]\,dx$ c) a) $\int \frac{e^{\text{Cos}[x]}}{\text{Sin}[x]}\,dx$ c) $\int \text{Sin}[x]\,e^{\text{Cos}[x]}\,dx$

S $u = \ln(x)$ S or N $u = x^2$ will simplify to an easy integration by parts N S $u = \cos(x)$

- **Solve completely (right or wrong)**

$\int x^2\,\text{Log}[x^3]\,dx$ $u = x^3$ then $\frac{du}{dx} = 3x^2$, $\frac{du}{3x^2} = dx$

$$\int x^2 \ln(x^3)\,dx = \int x^2 \ln(u)\frac{1}{3x^2}\,du = \int \frac{1}{3}\ln(u)\,dx = \frac{1}{3}\left(u\ln(u) - u\right)$$

$$= \frac{1}{3}\left(x^3\ln(x^3) - x^3\right) + C$$

- **Limits of Integration, Solve completely: (right or wrong)**

$\int_0^1 x\,\text{Cos}[x^2]\,dx$ Let $u = x^2$ then $\frac{du}{dx} = 2x$, and $\frac{1}{2x}du = dx$

$$\int x\cos(x^2)\,dx = \int x\cos(u)\frac{1}{2x}\,du = \int \frac{1}{2}\cos(u) = \frac{1}{2}\sin(u) = \frac{1}{2}\sin(x^2) + C$$

$$\int_0^1 x\cos(x^2)\,dx = \frac{1}{2}\sin(x^2)\Big|_0^1 = \frac{1}{2}\sin(1) - 0$$

- **Thought/Concept questions**

Integration by Substitution is related to which rule for differentiation?

a) Sum Rule b) Product rule c) Chain Rule d) Quotient Rule

Given the functions g(x) and f(x), then: $\frac{d}{dx} f(g(x)) = f'(g(x) * g'(x)$ which means that: $f(g(x)) = ??$ $\int f'(g(x))g'(x)\,dx$

- **Integration by Parts**

Integration by parts is related to which rule for differentiation?

a) Sum Rule b) Product rule c) Chain Rule d) Quotient Rule

Given the functions g(x) and f(x), then:

$\frac{d}{dx} u(x)\,v(x) = u'(x)\,v(x) + u(x)\,v'(x)$

$u(x)\,v(x) = \int u'(x)\,v(x)\,dx + \int u(x)\,v'(x)\,dx$

and thus: $\int u(x)\,v'(x)\,dx = ???$ $u(x)\,v(x) - \int u'(x)v(x)\,dx$

- **Given the integrals below, put a BP by the integral if By Parts is appropriate, and N if it is not. If by parts is appropriate identify u , and dv , but do not solve ?**

a) a) $\int x \operatorname{Cos}[x]\, dx$ b) $\int x^5 \operatorname{Cos}[x^6]\, dx$ c) a) $\int x^2 e^x\, dx$ c) $\int \operatorname{Log}[x]\, dx$

a) BP, $u = x$ $\quad dv = \cos(x)\,dx$

b) N

c) BP, $u = x^2$, $dv = e^x\,dx$

c) BP, $u = \ln(x)$, $dv = dx$

- **Solve completely (right or wrong)**

$\int x \operatorname{Cos}[x]\, dx =$ Let $u = x$ $\quad v = \sin(x)$

$du = dx \quad dv = \cos(x)\,dx$

$$= x\sin(x) - \int \sin(x)\,dx = x\sin(x) + \cos(x) + C$$

- **Limits of Integration, Solve completely given that: (right or wrong)**

$\int_0^{\pi} x \operatorname{Cos}[x]\, dx =$

$$(x\sin(x) + \cos(x))\Big|_0^{\pi} = (\pi\sin\pi + \cos\pi) - (0\sin(0) + \cos(0))$$

$$= -1 - 1 = -2$$

- **Partial Fractions**

Integration by partial fractions is an algebraic trick that allows us to simplify the integrand of some integrals?

- **Given the integrals below, put a PF by the integral if Partial Fractions is appropriate, and N if it is not. If Partial Fractions express the partial fraction decomposition in terms of the unkown coeffients (do not solve for the coefficients) ?**

a) $\int \frac{x^3-2x^2-16}{x}\, dx$ b) $\int \frac{2x+3}{x^3-2x^2+1x}\, dx$ c) a) $\int \frac{x^2}{x^3-1}\, dx$ c) $\int \frac{2}{(x-2)(x-1)^2}\, dx$

a) N

b) PF

Note $x(x^2-2x+1) = x(x-1)^2$

so $\frac{2x+3}{x^3-2x^2+x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

c) N

c) PF

$\frac{2}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

- **Solve for the coefficents if: (right or wrong)**

$\frac{4x+1}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{\cancel{(x+2)}\ (x+3)}$

$4x+1 = A(x+3) + B(x-1)$

if $x = -3$ then $-11 = -4B$ so $B = \frac{11}{4}$

if $x = 1$ then $5 = 4A$ so $A = \frac{5}{4}$

- **Solve completely (right or wrong)**

Note $\frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

$$\int \frac{1}{(x-2)\ (x-3)}\, dx =$$

$1 = (x-3)A + B(x-2)$

if $x=3$ then $1 = B$

if $x=2$ then $1 = -A$

$$\int \frac{1}{(x-2)(x-3)}\,dx = \int \frac{-1}{x-2} + \frac{1}{x-3}\,dx$$

$$= -\ln(|x-2|) + \ln(|x-3|) + C$$

- **Numerical Techniques**

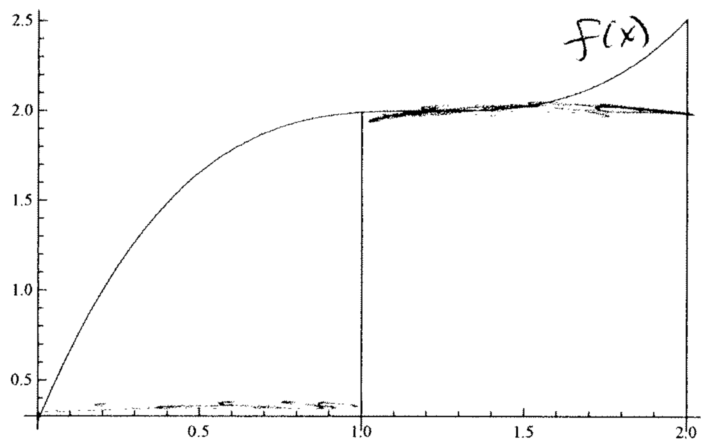
~~Which~~ of the following are reasons for using numerical integration techniques (select all that apply)?

a) Integral may not have an elementary anti-derivative

~~b)~~ Numerical techniques are more accurate than analytic techniques

c) Numerical techniques may be faster than analytic techniques on computers

- **Given the picture, answer the following questions**



Give the height of the first box using the following approximations.

a) LHS $f(0)$

b) RHS $f(1)$

c) MID $f(.5)$

d) TRAP $\frac{f(0)+f(1)}{2}$

What is the width of each box? 1

Draw and label the graphical representation of each technique (LHS, RHS, MID, TRAP) in the diagram. Too Messy see Text

In the first box, state whether the following approximations are over estimates, under estimates, or exact values.

a) LHS Under

b) RHS Over

c) MID Over

d) TRAP Under

- **Given the data below, answer the following questions:**

distance	0.	0.5	1.	1.5	2.
height	0.272	1.657	1.992	2.027	2.512

a) Approximate the value of the integral using the LSum technique

$(.272).5 + (1.657).5 + (1.992).5 + (2.027).5 =$

b) Why can't we use the midpoint approximation for this problem? Don't know the height at $x = .25$.

- **Error**

The error in the LHS estimate is proportion to $\frac{1}{n}$, what are the errors proportional to for the following techniques:

a) RHS $\frac{1}{n}$

b) Mid $\frac{1}{n^2}$

c) Simp $\frac{1}{n^4}$

If the error in the Trap method is proportion to $\frac{1}{n^2}$, and we double the number of step, then the error is reduced by what factor. i.e. new error = $\frac{\text{old error}}{?}$

$$\text{new error} = \frac{\text{old error}}{4}$$

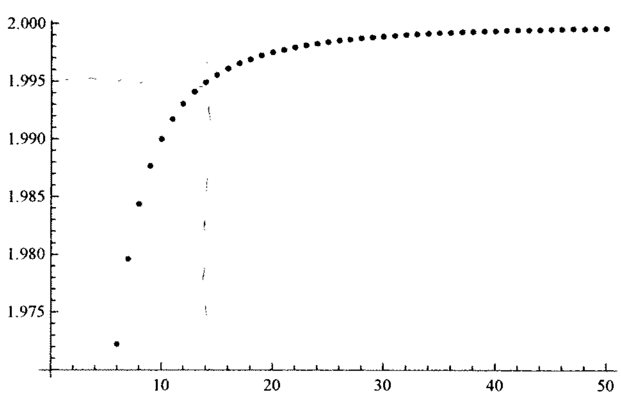
Given the graph below that shows the trap approximation and different step numbers,

a) What do think the exact value of the integral is? I would expect ~~Iat~~ 2

Note this does not garuntee an exact value of 2.

b) If i wanted an approximation that was good to two decimal places, about how many steps should i be taking?

$n \geq 14$



- **Improper Integrals**

Which of the following are reasons why an integral would be improper? **this could be T or F, or list the two reasons**

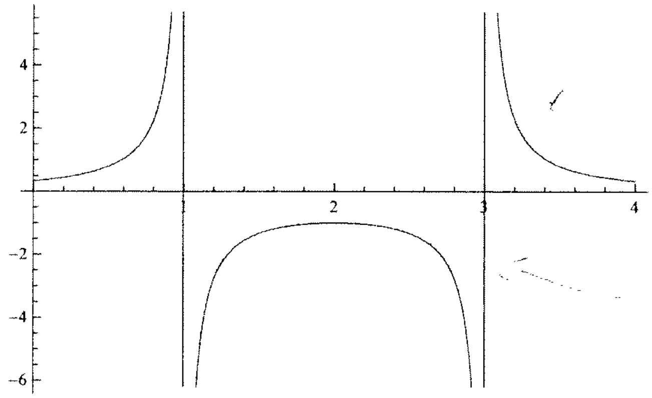
a) Integrand becomes unbounded

b) Integral may not have an elementary anti-derivative

c) Limits of integration go to infinity

d) Numerical techniques don't converge

- **Given the picture of the function g[x] shown below, answer the following questions**



$g(x)$

vertical asymptote

Label each of the following integrals as proper or improper.

a) $\int_0^{0.5} g[x]\, dx$ Proper

b) $\int_{0.5}^{1.5} g[x]\, dx$ Improper

c) $\int_{1.5}^{2.5} g[x]\, dx$ Proper

d) $\int_{2.5}^{3.5} g[x]\, dx$ Improper

e) $\int_{3.5}^{\infty} g[x]\, dx$ Improper

Label each of the following integrals as proper or improper.

a) $\int_0^{10} \frac{1}{x-8}\, dx$ Improper

b) $\int_2^{3} \frac{x^2-3}{x-1}\, dx$ Proper

c) $\int_1^{5} e^{x-5}\, dx$ Proper

e) $\int_1^{\infty} e^{x-5}\, dx$ Improper

Given that the $\int x^{-2}\, dx = \frac{-1}{x} + C$,

a) Re-write the following improper integral $\int_0^1 x^{-2}\, dx$ as the limit of a proper integral.

$$\int_0^1 x^{-2} dx = \lim_{a \to 0^+} \int_a^1 \frac{1}{x^2} dx$$

b) Evaluate the limit, and give the solution to the integral

$$= \lim_{a \to 0^+} -x^{-1} \Big|_a^1$$

$$= \lim_{a \to 0^+} \left(-1 + \frac{1}{a}\right)$$

$$\Rightarrow \text{DNE}$$