

Quiz 2

Name: *Key*

Directions: Answer each question to the best of your ability. You may use a calculator, but show all work to earn partial credit. The value of each question follows the question.

1. Let $\vec{u} = 2\vec{i} + 3\vec{j} + \vec{k}$, $\vec{v} = -\vec{i} + 2\vec{j} - 3\vec{k}$, and $\vec{w} = 2\vec{i} - \vec{j} - \vec{k}$. For each pair of vectors determine whether the angle between them is less than, equal to, or greater than 90 degrees by calculating the dot products. (3 pts each)

$\vec{u} \cdot \vec{v}$	$\vec{w} \cdot \vec{v}$	$\vec{u} \cdot \vec{w}$
$(2 \cdot -1) + (3 \cdot 2) + (1 \cdot -3)$	$(2 \cdot -1) + (2 \cdot -1) + (-3 \cdot -1)$	$(2 \cdot 2) + (-3 \cdot 1) + (-1 \cdot -1)$
1	-1	0
$< 90^\circ$	$> 90^\circ$	90°

2. For which value(s) of λ makes the following 2 vectors perpendicular: $\vec{u} = \lambda\vec{i} + 3\vec{j} + \vec{k}$, and $\vec{v} = \lambda\vec{i} + 2\vec{j} - 7\lambda\vec{k}$ (4 pts)

By dot product: $\lambda^2 + 6 - 7\lambda = 0$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda = 6, 1$$

3. Calculate the projection of \vec{v} onto \vec{u} if $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$, and $\vec{v} = -\vec{i} + 2\vec{j} - 3\vec{k}$. (4 pts)

$$\begin{aligned} \vec{v} &= \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} (\vec{u}) = \frac{-2 + 4 - 3}{4 + 4 + 1} (\vec{u}) \\ &= -\frac{1}{9} (\vec{u}) = -\frac{2}{9} \vec{i} - \frac{2}{9} \vec{j} - \frac{1}{9} \vec{k} \end{aligned}$$

4. Calculate $\vec{u} \times \vec{v}$. (4 pts)

$$\begin{aligned} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ -1 & 2 & -3 \end{vmatrix} &= (-6-2)\vec{i} - (-6+1)\vec{j} + (4+2)\vec{k} \\ &= -8\vec{i} + 5\vec{j} + 6\vec{k} \end{aligned}$$

5. Calculate the equation of the plane containing the points (0,-2,2), (1,0,1), and (2,6,4) (4 pts)

$$\begin{aligned} \vec{v}_1 &= \langle 1, 2, -1 \rangle \\ \vec{v}_2 &= \langle 2, 8, 2 \rangle \end{aligned}$$

$$\begin{aligned} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ 2 & 8 & 2 \end{vmatrix} &= (4+8)\vec{i} - (2+2)\vec{j} + (8-4)\vec{k} \\ &= 12\vec{i} - 4\vec{j} + 4\vec{k} \rightarrow 12x - 4y + 4z = 16 \\ &\quad \downarrow \text{OR} \end{aligned}$$

$$3x - y + z = 4$$